Positive Impact of Isomorphic Changes in the Environment on Collective Decision-Making

Palina Bartashevich
Otto-von-Guericke University, Magdeburg, Germany
palina.bartashevich@ovgu.de

Sanaz Mostaghim
Otto-von-Guericke University, Magdeburg, Germany
sanaz.mostaghim@ovgu.de

ABSTRACT

The performance of self-organized collective decision-making systems highly depends on the interactions with the environment. The environmental bias factors can introduce indirect modifications in the behaviour of such systems, however, not all changes are for the worse. In this paper, we show how the isomorphic changes in the environment can improve the performance of the collective decision-making strategies, mostly used in the current state-of-the-art swarm robotics research. The idea is based on the usage of a special kind of an equivalence relation, namely isomorphism, which provides local changes in the environment while preserving the global information. The obtained results indicate that the isomorphic transformations, sharing a certain structure of the environment, can significantly accelerate the consensus time without compromising correctness of the final decision.

CCS CONCEPTS

• **Computing methodologies → Distributed artificial intelligence**; *Multi-agent systems*; *Intelligent agents*;

KEYWORDS

Collective decision making, collective perception, multi-agent systems, isomorphism, permutations

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Introduction. The behavior of self-organizing systems can be modified either by explicitly adjusting the behavior of the agents or by changing the environment. However, since it is not always possible to obtain a direct control over the individuals, their reactions to some environmental stimuli are triggered [2]. The goal of this paper is to work on tailored changes in the environment which will impose a positive impact on the collective behavior. To study this, we use a special kind of transformations, namely isomorphism relation [4], which is mostly used in mathematical and biological studies to gain a better understanding of equally related objects. These changes keep the global structure of the object (environment in our case) while only locally changing it. We especially investigate the influence of such changes on the behavior of self-organizing collective decision-making systems (further denoted as CDM). As the case study, we consider the Collective Perception scenario with a squared grid of $k \times k$ cells as the environment, where the cells are painted in $n = 2$ colors. The task for a swarm is to collectively decide which color out of n in the grid is a prevailing one. Basically, collective perception can be also considered as a mental process, which is happening in the human brains while looking at certain pictures, where agents represent the neurons of the prefrontal cortex [5]. There is no doubt that keeping always the same ratio of the colors but changing their spatial arrangements might confuse the observer (CDM in our case), as certain visual patterns lead to faster decisions than the others. That is, it is much easier to come up with a decision about prevailing color if the colors are already clustered in some groups, than if they are scattered randomly [5]. However, our experiments show that this is not the case for the existing CDM strategies. The results indicate that providing isomorphic changes in the environment positively influences the collective behavior in CDM and can be used as an effective tool to significantly improve the CDM's performance on difficult scenarios.

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Preliminaries. In the following, we introduce some mathematical definitions and concepts from Beth et al. [1], which provide the basis for the proposed methodology.

Definition 1. An **incidence structure** (or a **design**) is an ordered triple $(\mathcal{P}, \mathcal{B}, \mathcal{I})$, where $\mathcal{P}$ is a set of points, $\mathcal{B}$ is a set of blocks (where any block $B \in \mathcal{B}$, $B \subseteq \mathcal{P}$) , $\mathcal{I} \subseteq \mathcal{P} \times \mathcal{B}$ is an incidence relation between $\mathcal{P}$ and $\mathcal{B}$.

Definition 2. Let $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ be an incidence structure with points $\mathcal{P} = \{p_1, ..., p_k\}$ and blocks $\mathcal{B} = \{B_1, ..., B_r\}$. An **incidence matrix** $M = (m_{ij})$ for $\mathcal{D}$ is a $k \times r$ matrix defined by $m_{ij} = 1$ if $p_i \in B_j$ (i.e. p_i is incident with B_j, that is, $p_i \mathcal{I} B_j$) and $m_{ij} = 0$ otherwise.

A key tool to determine the similarity between two incidence structures is an **isomorphism relation**, which can be interpreted in terms of their incidence matrices in the following way.

Definition 3. Two incidence structures $(\mathcal{P}, \mathcal{B}, \mathcal{I})$ and $(\mathcal{P}', \mathcal{B}', \mathcal{I}')$ with incidence matrices M and M' are called **isomorphic** if and only if there exist permutation matrices P and Q (square binary matrices each row and column of which contain exactly only one '1') such that $PMQ = M'$. Here, H permutes the rows (i.e. points) and Q permutes the columns (i.e. blocks).

As an example of an incidence's relation measure, which remains unchangeable under transformations (i.e. invariant), one can choose the Smith normal form (SNF) [3]. That is, designs with different SNFs are non-isomorphic.

Representation of the Environment. The environment for the collective perception scenario is usually performed by a randomly generated square grid of black and white cells [7]. In order to consider different distributions of the colors, we identify the grid with an incidence structure. For this purpose, we perform the incidence structure as a matrix $M \in \mathbb{Z}_2^{k \times k}$, which describes the relationship between the columns (set of blocks: $\mathcal{E} = \{e_1, e_2, ..., e_k\}$) and the rows (points: $\mathcal{V} = \{v_1, v_2, ..., v_k\}$) of the grid (see Fig. 1). For every pair of indices (i, j), we have $v_i \mathcal{I} e_j$ and, hence, $m_{ij} = 1$, if and only if the corresponding cell c_{ij} is black colored.

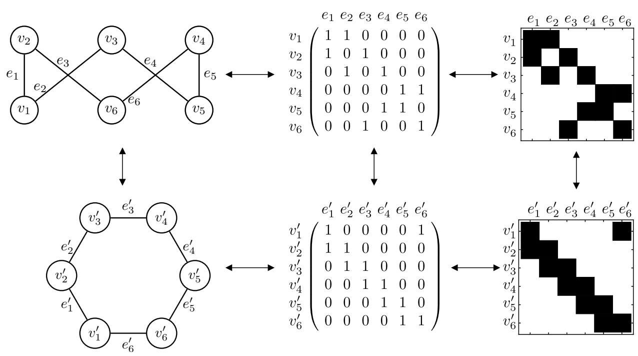

Figure 1: Different representations of isomorphic incidence structures. Example: Undirected graph ↔ Incidence matrix of a graph ↔ Grid of cells.

Having a matrix representation, we can introduce the isomorphism relationship on the grid. In order to create an isomorphic grid, M_{new}, to the given one M, i.e. $M_{new} \cong M$, $M_{new} \neq M$, we generate two not identical random permutation matrices $P, Q \in \mathbb{Z}_2^{k \times k}$, such that $M_{new} = P \cdot M \cdot Q$. Note that such changes will not affect the incidence relation on the grid but only its visual representation, keeping the global information unchangeable.

Although, it is very difficult to identify that two incidence structures are isomorphic [6], one can provide a numerical evidence that two of them are non-isomorphic. The procedure of generating a non-isomorphic matrix, M_{new}, from a given one, M, is based on the check that their SNFs are not equal. $SNF(M)$ returns a diagonal matrix $S \in \mathbb{Z}^{k \times k}$ along with two unimodular transformation matrices $U, V \in \mathbb{Z}^{k \times k}$, such that $S = U \cdot M \cdot V$. If $S_{new} \neq S$, then $M_{new} \ncong M$. A new non-isomorphic matrix can be additionally obtained by the substitution of some '0' values with '1's (or vice versa). However, such changes will cause the destruction of the old incidence relations and setting of the new ones, thereby modifying the global information in the environment (i.e. the proportion of the colors). This type of change covers the case considered in [7] and therefore is not the focus of our study.

Experiments. The goal of the experiments is to investigate the impact of the isomorphic transformations in the environment on the performance of the primarily used CDMs in [7], i.e. the *majority rule* (DMMD), the *voter model* (DMVD) and the *direct comparison* (DC). To validate their performance, we consider two commonly used metrics: (1) *Exit probability* (E_N) to measure the ratio of successful runs among all simulations and (2) *Consensus time* ($T_N^{correct}$) as the number of iterations until all the agents converge to the correct opinion. We perform a multi-agent simulation similar to the one described in [7], with 40 simulation runs maximum of 400 sec each, for each type of the task difficulty ρ_b^*. As the task difficulty, the proportion of the colors is considered.

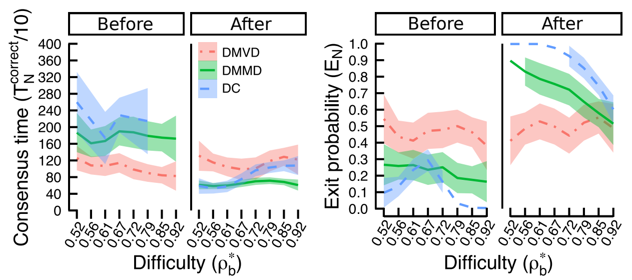

Figure 2: Consensus time ($T_N^{correct}$) and exit probability (E_N) "before" and "after" applying isomorphism for eight types of the task difficulty ρ_b^*.

Results and Conclusion. The results (see Fig. 2) show a clear significant difference (Mann-Whitney U-test, $p < 0.05$) in the performance for the DMMD and DC in both metrics, $T_N^{correct}$ and E_N, before and after applying the isomorphism on one of the most difficult visual pattern, represented by clustered vertical/horizontal lines, creating a stripe. The performance of the DMVD strategy is the best one among the others in the "before" case, followed by the DMMD, while the DC shows its the worst performance (completely fails for $\rho_b^* > 0.79$). However, only the DMVD does not improve the performance in the "after" environment, staying at the chance level. In a nutshell, by using isomorphic changes, we are able to map complicated patterns from the environment into "simpler" ones without changing their original structures. In this way, the CDMs can reach a consensus in a faster and more accurate way than in the initial environment. Although the presented study is limited by the possibility to provide direct changes in the environment, we believe that the presented finding deepens our understanding to develop better CDM strategies and, as part of the future work, we are going to consider an isomorphism relative to the agents' minds.

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