



Benchmarking Collective Perception: New Task Difficulty Metrics for Collective Decision-Making

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Abstract. This paper presents nine different visual patterns for a Collective Perception scenario as new benchmark problems, which can be used for the future development of more efficient collective decision-making strategies. The experiments using isomorphism and three of the well-studied collective decision-making mechanisms are conducted to validate the performance of the new scenarios. The results on a diverse set of problems show that the real task difficulty lies not only in the quantity ratio of the features in the environment but also in their distributions and the clustering levels. Given this, two new metrics for the difficulty of the task are additionally proposed and evaluated on the provided set of benchmarks.

Keywords: Collective decision making · Collective perception · Benchmarking · Multi-agent systems · Isomorphism

1 Introduction

In the past decade, researchers have shown an increased interest in the study of self-organizing collective decision-making (further denoted as CDM) using multi-agent and swarm robotics systems. There are two typical scenarios to test the performance of such systems, Site Selection and Collective Perception [9, 13]. However, unlike in optimization, there are no standard benchmark functions (artificial landscapes) for these scenarios. In this paper, we intend to propose a new set of benchmarks, particularly for Collective Perception, along with two new metrics to quantify the *difficulty of the task* for more comprehensive validation and comparison of new CDM methodologies.

We focus on the Collective Perception [14], where the individuals move in an environment with a certain pattern and make a collective decision about the specific features derived from this pattern. In particular, in this paper, the final collective decision has to be taken on which color is prevalent in the scene.

The Site Selection mechanisms [9] are slightly different. There, the individuals start their movements from a certain region in the environment, the “nest”, explore alternative sites for other opinions and go back to the “nest” to exchange the new explored opinions with other individuals. Although the latter is mostly observed in nature, i.e. the behaviour of honeybees and ants, multiple returns

to the “nest” cost extra time and energy, which are ones of the most critical limitations in the robotic systems. While in the collective perception scenario the agents are able to act and decide directly “on the fly” during the exploration without going back to the starting point. Among possible applications are the monitoring of the air and water quality, the concentration estimate of CO₂ and oxygen in the burning buildings, or the frequency evaluation of the scattered natural resources in the hard-to-reach areas.

The most common benchmark scenario for collective perception is a randomly generated 2D square grid, mostly equiprobably painted with two colors, black and white [12, 14]. In this paper, we aim to develop new benchmark problems representing various features which can be used as a baseline for further validation of the CDM strategies. Our benchmark contains a set of nine different patterns, taken from the matrix visualization literature [2–4]. Unlike the existing Collective Perception scenario with random distribution of the features [8, 14], these visual patterns contain certain structural information, i.e. clustering, which can cover a diverse representation of various real-world scenarios. In order to validate the proposed benchmarks, we investigate the influence of different feature distributions on the performance of the existing CDM methods, which are primarily used in the current state-of-the-art [13], the *majority rule* (DMMD), the *voter model* (DMVD) and the *direct comparison* (DC).

In order to verify how general is the proposed benchmarking approach, we refer to the recent study [1], which has shown a positive impact of a special kind transformation in the environment, namely isomorphism, on the CDM performance. Different from [1], here, we use isomorphism to prove that the color-ratio difficulty, mostly examined in the previous research, is an intrinsic property of a certain CDM strategy, while the clustering level of the features is the actual *difficulty of the task*. To support this claim, we introduce two new metrics to specify the task difficulty for the CDM and perform the analysis of the proposed benchmarks based on them.

The paper is organized as follows. In Sect. 2, we provide the related work on collective perception. Afterwards, we describe new task difficulty metrics along with the multi-agent simulation in Sect. 3. Section 4 presents the evaluation of the obtained results and the paper is concluded in Sect. 5 including some discussion.

2 Related Work

The Collective Perception scenario was originally introduced in 2010 by Morlino et al. [8], who studied how a swarm of robots can collectively encode the density of black spots on the ground using flashing signals. In [14], Valentini et al. compared the performance of several CDM strategies in the context of Collective Perception, considering two black-and-white static grid setups: the easiest one with the proportion of the prevailing color closely twice of another, and the most difficult one, characterized by almost equal color ratio. Later, Strobel et al. in [12] tested the same CDM approaches on more percentage variations. In [6], Ebert et al. investigated for the first time a 3-feature case and demonstrated

a new decentralized decision-making algorithm with a dynamic task allocation strategy to classify a 3-colored grid. Prasetyo et al. in [10] has considered an application of a CDM to the dynamic Site Selection problem, where the quality of two sites is changing over time. Recently, Valentini has also published a book [13] about the design and analysis of CDM strategies for the best-of- n problems. However, in the application part, attention mainly for $n = 2$ is paid. So far, all the research in this area has studied either the quality or the total amount of the features in the environment (i.e. global information), as a measure for the *task difficulty*, ignoring their actual distributions.

3 Methodology

For the generation of benchmark environments, we refer to the matrix visualization literature [2–4] and consider nine of the most important visual patterns, which can be observed in visual matrices (see the top of Fig. 2). In order to prevent the attachment of the further simulation results to the concrete configuration, for each type of the pattern and the color-ratio, a random environment with the same visual structure as the whole class corresponding to this pattern is generated. For some color ratios (e.g. 48% black and 52% white, see top Fig. 2), the patterns may contain some artifacts, due to the insufficient amount of available cells to shape the pattern. That is, random black cells placed out of the pattern or, vice a verse, white cells disrupting the pattern can be observed.

3.1 Pattern Metrics

In order to classify the considered patterns, we propose the following metrics:

- *Entropy* (E_c): It measures the density of the clusters in pattern P .

To calculate E_c , we need to detect clusters C_i in P . Without loss of generality, we assume that black color determines the pattern and, hence, clustering. Therefore, let $|C_i|$ denotes the number of black cells that belong to the cluster C_i and N_{bl} is the total number of black cells in P . Then, the metric E_c is defined as follows:

$$E_c = \frac{H_c^{max} - H_c}{H_c^{max}} \quad (1)$$

$$H_c = - \sum_{i=1}^M \frac{|C_i|}{N_{bl}} \log_2 \frac{|C_i|}{N_{bl}} \quad (2)$$

$$H_c^{max} = - \sum_{N_{bl}} \frac{1}{N_{bl}} \log_2 \frac{1}{N_{bl}}, \quad (3)$$

where M is the number of clusters, H_c^{max} is the maximum entropy. The value of E_c increases with decreasing number of clusters ($E_c \rightarrow 1$ is ordered, $E_c \rightarrow 0$ is random). However, the value of E_c does not characterize how this cluster (or clusters) looks like, i.e. is it compact (e.g. as a “block”) or more sparse (e.g. as a “chain”). To address this issue, we also consider the following metric:

- *Moran Index (MI)*: It estimates the level of connectivity between clusters (if any) in pattern P .

In our case, MI is calculated as the correlation of the colors between adjacent cells c_i and c_j in P , where N is the total number of cells:

$$MI = \frac{N}{\sum_{ij} w_{ij}} \cdot \frac{\sum_{ij}^N w_{ij} (c_i - \bar{c})(c_j - \bar{c})}{\sum_i^N (c_i - \bar{c})^2}. \quad (4)$$

The value of c_i is equal to 1 if it is black, and 0 if it is white; \bar{c} is the mean of all cell values in P and $\bar{c} = N_{bl}/N$. The values of w_{ij} , $w_{ij} = w_{ji}$, define the the degree of spatial closeness of the cells c_i and c_j , i.e. $w_{ij} = 1$ if the cells c_i and c_j have a common side, i.e. placed as \dagger (where \dagger shows which cells neighbour a central one), otherwise $w_{ij} = 0$. The more \dagger -connected black cells are in a pattern, the closer the value of MI is to 1. When colors tend to be more randomly distributed, then $MI \rightarrow 0$ (i.e. random pattern gives $MI = 0$). Alternation of white and black (like in a chessboard) brings $MI \rightarrow -1$.

- *Color Ratio*: It describes the “task difficulty” from [12,13].

It is calculated as $\rho_b^* = \frac{N_{bl}}{N_{wh}}$, $\rho_b^* \in [0, 1]$, taking into account that $N_{wh} > N_{bl}$. The more visible is the difference between the amount of colors, the lower is the value of ρ_b^* (e.g. 34% – 66%, $\rho_b^* \approx 0.52$). As soon as the proportions of the colors are coming closer to each other, $\rho_b^* \rightarrow 1$ (e.g. 48% – 52%, $\rho_b^* \approx 0.92$).

3.2 Multi-agent Simulation

To conduct the experiments, we implement a multi-agent simulation along with nine pattern generators in MatlabR2017a. The environment is defined by a square grid of 20×20 cells, 1×1 unit each, painted over in black and white. Without loss of generality, we consider that in all the considered environments the white color is prevailing. As in the previous research [12], we consider 100 iterations in simulation as 1 s, and we plot the simulation environment each 10 iterations (i.e. 0.1 s). We use a swarm of 20 agents, initially assigned with half for opinion white and half for black, keeping the other agents parameters similar to Valentini et al.’s work [15]. Each agent can be in one of the two alternating states: (1) *exploration*, where it moves and only estimates the quality of its current opinion, or (2) *dissemination*, when it moves and only exchanges its own opinion with the others. At the end of the state (2), it makes a decision on either to keep or to switch its current opinion. The communication between agents is set only pairwise for each 0.1 s in a random order within the distance of 5 units and only if both of them are in states (2). Each agent logs the opinions of its neighbors during the last 0.3 s of its (2) state and takes the last 2 opinions to decide based on one of the mentioned in Sect. 1 decision-making strategies (further denoted as DMs). Two metrics, *Exit probability* (E_N) and *Consensus time* ($T_N^{correct}$), are used to evaluate the performance of DMs. E_N measures the ratio of successful runs among all simulations. The simulation is considered successful, if the collective reached consensus on the correct option. $T_N^{correct}$ defines the number of iterations until all the individuals come up with the same opinion.

4 Experimental Study

4.1 Pattern Characteristics

In order to determine how the patterns quantitatively differ from each other, we evaluate the mean values and the standard deviations of *Entropy* and *Moran Index* over 100 generations of each pattern (see Fig. 1-left). According to the preliminary calculations, the metrics' values mainly differ only within a pattern type and are not affected by the change of the color ratio (as intended). The Kruskal-Wallis ANOVA analysis of *MI* values reveals that there is a significant difference ($p < 0.01$) between all the grid-patterns, except for “*Off-diagonal*” and “*Block*”. Referring to the same analysis of the E_c values, we group the patterns correspondingly to their non-significant differences with the others (see Fig. 1): (1) “*Off-diagonal*” and “*Block*” (in blue); (2) “*Stripe*” and “*Band-Random-Width*” (in green); (3) “*Band-Stripe*”, “*Bandwidth*” and “*Star*” (in red). Therefore, patterns belonging to the same group have a non-significant difference ($p > 0.01$) in distributions of their E_c values. However, all of them are significantly different ($p < 0.01$) according to their Moran indexes, except of the first group. “*Band*” and “*Random*” do not belong to any of the formed groups and are significantly different ($p < 0.01$) from all the others in both parameters. We expect similar results in the CDMs performance on the patterns within the same group.

4.2 Experiment I - Influence of the Patterns

The goal of the first experiment is to show the influence of different pattern configurations and their respective color ratios on the performance trend of CDMs. We perform 40 simulation runs with maximum 400 s each. That is, if the swarm was not able to reach the consensus during these 400 s or came to the wrong decision before the time expired, the simulation is stopped and the run is considered as unsuccessful.

Figure 2 shows the mean consensus time calculated only among successful runs and the exit probability obtained on nine different patterns for the three tested CDMs within eight variants of color ratio. The curves are created by local regression over boxplots (not reported here) with shading areas of 95% confidence interval. The DMVD strategy shows rather stable results for all “difficulties” (ρ_b^*) in both $T_N^{correct}$ and E_N on all the patterns (except for *P5-Stripe*), while the performance of DMMD and DC differs among the patterns. However, the exit probability of DMVD for the most of the patterns (i.e. P3-P9) is mainly observed by the chance level. *P1-Random* does not have any structure and was primarily the focus of the previous studies. We also obtained here the similar trend for the CDMs as in [12].

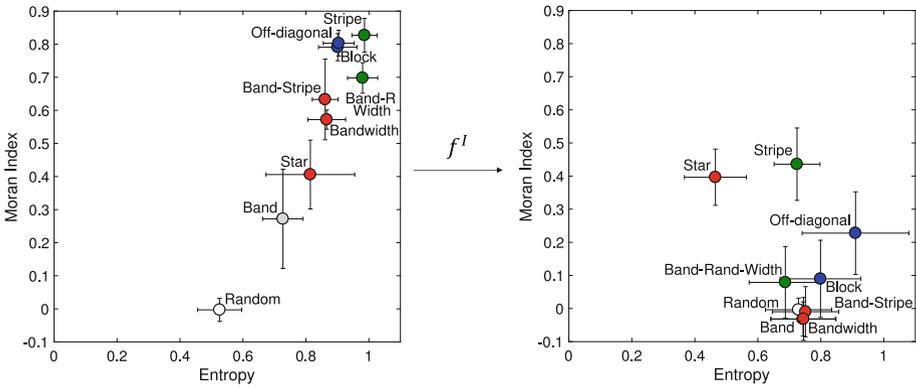


Fig. 1. Scatter diagram of the statistics for patterns characteristics (means (points) and standard deviations (bars)) “before” (left) and “after” applying isomorphism (right). Colors indicate the belonging to one and the same group. (Color figure online)

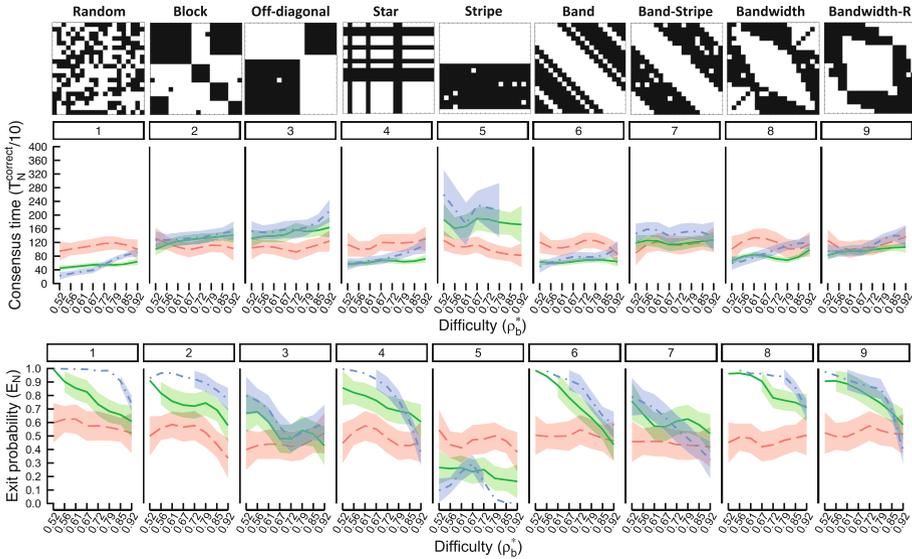


Fig. 2. Consensus time ($T_N^{correct}$) and exit probability (E_N) over eight types of the task difficulty ρ_b^* for each of nine patterns (top row). Dashed (red), solid (green) and dot-dashed (blue) lines correspond to the DMVD, DMMD and DC strategies respectively. (Color figure online)

Patterns P_4 -Star (performed by horizontal and vertical lines) and P_6 -Band (by diagonals of the upper and lower triangles) show similar results to P_1 -Random in $T_N^{correct}$ for all three CDM strategies. In addition, we obtain here the lowest value of consensus time for DMMD among all the other considered

patterns and DMs (of around 40 s), which is slightly increasing for higher ρ_b^* . Nevertheless, the $T_N^{correct}$ performance of DC strategy on P4 and P6 is worse than on P1, increasing in $T_N^{correct}$ and rapidly decreasing in E_N after $\rho_b^* \approx 0.72$. The E_N result for DMMD is similar and reaches the chance level by $\rho_b^* \approx 0.92$. The considered patterns, i.e. P1, P4 and P6, are characterized by $MI < 0.5$ and different values of E_c .

P8-Bandwidth (visual rhombus-like enclosure with fixed width around the main diagonal, $MI = 0.572 \pm 0.029, E_c = 0.866 \pm 0.06$) is “the second best” pattern after P1, P4 and P6, where DMMD is still one of the fastest strategies among the others with “ups and downs” in the performance between $\rho_b^* = [0.61, 0.79]$ and $\rho_b^* = (0.79, 0.92]$ respectively. DC, here, is characterized by the linear growth in $T_N^{correct}$ with increasing ρ_b^* . For $\rho_b^* \approx 0.92$ the DMs strategies perform actually the same (even with equal exit probabilities), while for other ρ_b^* the E_N resembles the similar trend as in P1 for all DMs.

In patterns *P9-Bandwidth-Rand* (the same as P8 but with random width, $MI = 0.698 \pm 0.046, E_c = 0.98 \pm 0.048$) and *P7-Band-Stripe* (the same as P6 but lines are clustered, $MI = 0.633 \pm 0.122, E_c = 0.861 \pm 0.041$), there is no single DM strategy which is better than the other one in terms of consensus time (DC even shows the highest $T_N^{correct}$ in P7 with the linearly decreasing E_N , getting lower than the chance level for $\rho_b^* > 0.79$). On P9, the E_N trend of DMMD and DC is similar.

P2-Block and *P3-Off-diagonal* (for both $MI \approx 0.8, E_c \approx 0.9$) are characterized by similar visual structures with the difference that in P3 there are only two rectangular coherent areas in the corners of off-diagonal, while in P2 the blocks must cover the whole main diagonal and only for higher ρ_b^* some possible blocks are added in the off-corners. The performance of DMMD strategy here is worse than DMVD and is not significantly better than DC for all ρ_b^* . In P3, DMMD and DC have higher $T_N^{correct}$ than in P2 along with non significantly different exit probabilities, which are lower than in P2, and both are characterized by the rapid decline to the chance level, which does not significantly change after $\rho_b^* > 0.72$.

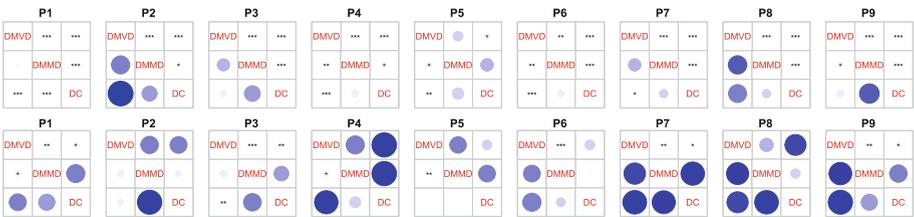


Fig. 3. The heatmaps of p -values (i.e. $*p < .05, **p < .01, ***p < .001$) according to the Mann-Whitney U-test indicating a statistically significant difference in the performance between DMs on each pattern. The lower triangular (LT) part of the matrix corresponds to the results “before” a nd the upper to the “after” applying isomorphism (top row: $\rho_b^* \approx 0.52$; bottom row: $\rho_b^* \approx 0.92$).

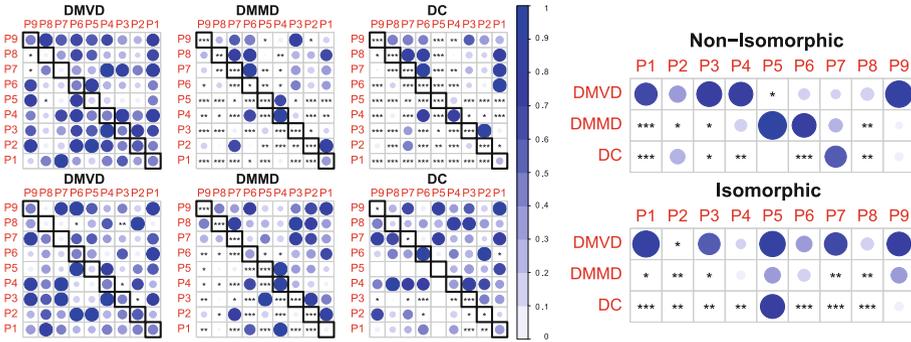


Fig. 4. The heatmaps of significance levels according to the Mann-Whitney U-test for the difference in the performance of the DMs within the patterns (on the left) and on one DM strategy with itself between the highest and the lowest values of ρ_b^* on each pattern (on the right). Left: The lower (LT) and the upper triangular (UT) parts of the matrix correspond to the results “before” and “after” isomorphic changes (top row: $\rho_b^* \approx 0.52$; bottom row: $\rho_b^* \approx 0.92$). The diagonal elements show how the performance of a certain DM strategy on one pattern changes after applying isomorphism. The size of the circles reflects the same value as the color intensity.

P5-Stripe (clustered vertical/horizontal lines, $MI = 0.827 \pm 0.051$, $E_c \rightarrow 1$) is a notable exception among the other patterns. The performance of DMVD strategy here is the best one among the others in both $T_N^{correct}$ and E_N with a slight decreasing of consensus time for higher ρ_b^* , followed by DMMD, while DC shows its the worst performance. The DC strategy completely fails for $\rho_b^* > 0.79$ (with $E_N \rightarrow 0$) and is characterized by the high variability for lower ρ_b^* with the lowest E_N than the other DMs. However, the exit probability for both DMMD and DC strategies here is less than by the chance.

Discussion. The obtained results confirm our initial claim that the real **task difficulty** lies not in the amount of prevailing feature (i.e. ρ_b^*) but in the distribution of the features. Our analysis shows that in the patterns with low density of clustering ($MI \leq 0.6$ and $E_c < 0.8$: P1, P4, P6, P8) the DMMD performs the best, while for the patterns with higher clusterization ($0.6 < MI \leq 0.8$ and $E_c \rightarrow 1$: P2, P3, P5, P7, P9) the DMMD performance significantly drops. P5 is the hardest pattern among all of the others ($MI > 0.8$, $E_c \rightarrow 1$), where the DMMD has the highest time with the lowest probability of reaching the consensus. The DC strategy shows mostly similar consensus time as DMMD within the patterns and the color-ratios but with higher variability and more rapidly decrease in the exit probability at higher ρ_b^* than the DMMD (in P4, P5, P7 and P9). It also completely fails in P5 (after $\rho_b^* > 0.79$). The results obtained by the DMVD in all of the considered patterns are mostly due to the chance. Figures 5 and 6 (both left) support the above conclusions.

4.3 Experiment II - Isomorphic Patterns

In order to evaluate the generalizability of the proposed benchmarks, in the second experiment, we use isomorphic transformations, firstly considered in [1]. Such transformations allow us easily to construct a large variety of structurally identical objects at the global level (i.e. environments with similar connectivity interactions and sharing the same amount of features). There, the grid of black-and-white cells is identified with a certain undirected graph via its incidence matrix [1], resembling the corresponding pattern with ‘0’s and ‘1’s instead of white and black colors. To construct an isomorphic environment to the given, we multiply an incidence matrix of the current environment, $M \in \mathbb{Z}_2^{k \times k}$, on two random permutation matrices, $P, Q \in \mathbb{Z}_2^{k \times k}$, as follows: $M' = PMQ$. Doing by this, we aim to investigate the performance of CDM even on a more diverse set of benchmark problems. In the following, we fix two extreme color ratios (the easiest and the hardest ones), namely $\rho_b^* \approx 0.52$ and $\rho_b^* \approx 0.92$, and study the influence of isomorphism on the consensus time and the exit probability within various patterns using different CDMs. Additionally, we also investigate how the isomorphic changes differ between the patterns.

The results show a clear difference in the performance of the CDM strategies in both metrics, $T_N^{correct}$ and E_N , before (Figs. 5 and 6-left) and after applying the isomorphism on the patterns (Figs. 5 and 6-right). The comparison is supported by Figs. 3 and 4 showing the levels of significant difference in the consensus time between the DMs, patterns and color-ratios, according to the Mann-Whitney U-test. From the data in Fig. 4, we can see that the DMVD strategy has no significant difference on its performance either within the patterns “before” or “after” the isomorphic changes nor within the “difficulties”. Along with Figs. 5 and 6, one can claim that it completely relies on a random chance. For the DMMD and DC strategies the picture is different. The DMMD and DC performances are significantly improved on isomorphic patterns with respect to the initial ones (see the diagonal elements in Fig. 4-left), and are characterized by a decrease in the consensus time and increase in the exit probability for both considered “difficulties” (Figs. 5 and 6). While the $T_N^{correct}$ values, obtained on the patterns before the transformation, mostly significantly differ between each other (see the LT-parts in Fig. 4), after transformations the differences in $T_N^{correct}$ within the patterns disappear (especially for $\rho_b^* \approx 0.92$, see the UT-parts in Fig. 4). However, for $\rho_b^* \approx 0.52$, *P4-Star* and *P5-Stripe* indicate a statistically significant increment in $T_N^{correct}$ among the other “after”-patterns but both are not statistically different between each other for both DMMD and DC (see P4-P5 columns and rows of the UT-parts in Fig. 4-left). Interestingly, the DMs strategies indicate a significantly different performance in the consensus time with each other on all “after”-patterns (except P5, where only DMVD and DC differ with $p < 0.05$) for $\rho_b^* \approx 0.52$ (see the UT-parts in the top row of Fig. 3). But this is not true for the “before” case (see the top row, the LT-parts in Fig. 3): there is even no significant differences between the DMMD and the DC on all the patterns (except P1). For $\rho_b^* \approx 0.92$ (see the bottom row, UT-parts in Fig. 3), the differences within all the DMs strategies on “after”- P2, *P4-Star*,

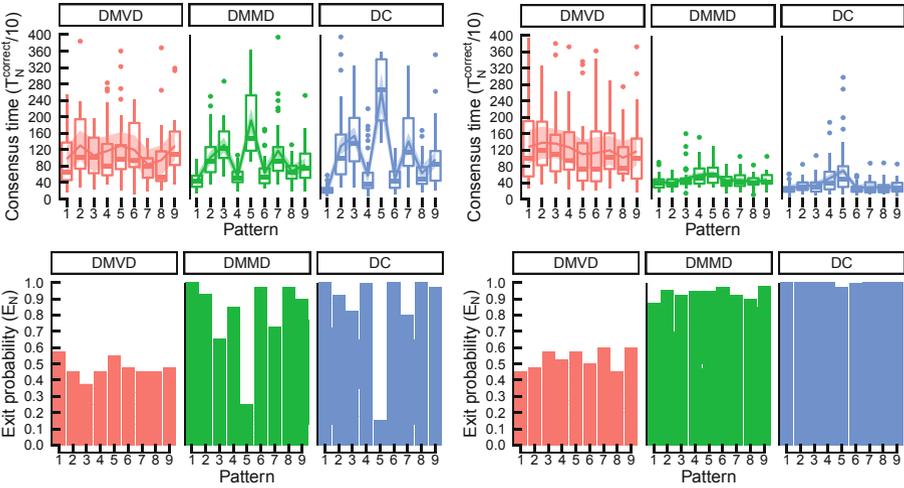


Fig. 5. Comparison of consensus time ($T_N^{correct}$) and exit probability (E_N) within patterns “before” (left) and “after” applying isomorphism (right) for the “task difficulty” of $\rho_b^* \approx 0.52$. The curves are fitted via local regression with shading areas representing 95% confidence interval.

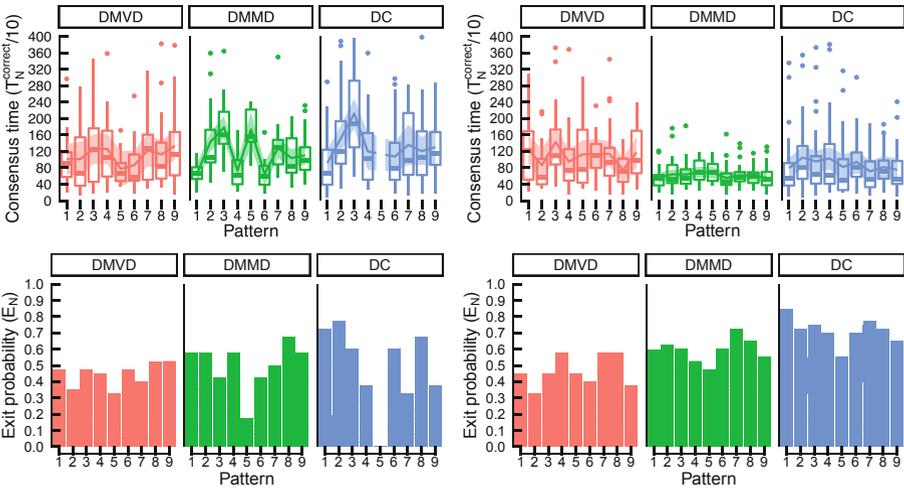


Fig. 6. The same comparison as in Fig. 5 but for the “task difficulty” of $\rho_b^* \approx 0.92$.

P5-Stripe and P8 are non-significant, and even on the other patterns there is no significant difference in the DMMD and the DC performance. However, the consensus time of the strategies itself has significantly decreased after isomorphism. In summary, the DC strategy yields the best performance on isomorphic patterns, followed by the DMMD and then the DMVD, regardless of the color

ratios. The analysis in Fig. 4-right shows that the isomorphic transformations do not considerably change the influence of the ρ_b^* -“difficulty” on the DMs. That is, the DMMD is significantly worsening on one and the same “before”- and “after”-patterns (i.e. P1-P3 and P8), while for DC this holds for all “before”- (except P2, P6 and P7) and “after”-patterns (except P5 and P9).

Discussion. Although an isomorphism preserves the global structure of the environment, the characteristics of the patterns with high density of clustering after applying isomorphic mappings have changed. From the data in Fig. 1, we can see that almost all “after”-patterns are characterized by $E_c \in [0.7, 0.8]$ and significantly decreased MI values (i.e. $MI \approx 0$), except for *P3-Off-diagonal*, *P4-Star* and *P5-Stripe*, where $MI \in [0.2, 0.5]$. Interestingly, exactly on these exceptions for both ρ_b^* (the easiest and the hardest) the analysis illustrates a significant increase in $T_N^{correct}$ w.r.t. other patterns. These findings support the results of our first experiment, indicating that the higher density of clustering significantly degrades the performance of CDM. To sum up, the difference in the performance of a certain CDM approach vanishes on isomorphic patterns and narrows to its intrinsic properties, thereby underlining the generalization and necessity of the proposed benchmarks.

5 Discussion and Conclusion

In this paper, we extended a Collective Perception scenario, used in the swarm robotics research, with nine different patterns to test the generalizability of the existing collective decision-making strategies and to promote the future development of the new ones. Previous research examined only a single and the easiest example of the environment, considering the proportion of the features as a “task difficulty” for decision makers. However, our benchmark study reveals that the “difficulty” connected with the ratio of the colors (i.e. global information) is mainly related to the intrinsic property of a certain CDM method, while the distribution of the features in the environment (i.e. local information) is the actual *difficulty of the task*. In this scope, we also proposed and examined two new metrics that have shown to be a good choice to define the difficulty and to predict the behavior of the CDM. Experiments on isomorphic test problems were also conducted to support the study. Isomorphism has been already proven to be an effective tool to design a diverse set of digital cognitive games [11]. Similar, in this work, we used isomorphism to diversify the set of the benchmarks. At the same time, our results fully support the previous research [1] on a bigger set of problems, where isomorphic changes in the environment result in the speed up of CDM. From this side, a collective perception is considered as a cognitive activity mediated in human brains. There, isomorphism is described as the brain moving objects in order to facilitate a decision-making process. While for humans it is more relevant to mentally re-order the objects to enhance the problem solving [5] (e.g. to figure out the prevailing color), to make direct changes in the real environment with acting swarm of robots seems to be problematic. However, if the agents will be able to build individually or collectively a cognitive map of the

environment during the exploration process (i.e. latent learning [7]), isomorphic transformations can be used inside their “inner world” [16] to assist a CDM, without the need to be adapted to the specific scenario.

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