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Tutorials for EMO Lecture Summer Semester 2024

Assignment 1 Meta-heuristic Optimization and Pareto Dominance

Please answer the following questions regarding meta-heuristic optimization and Pareto-dominance.

- What is the difference between a heuristic and a meta-heuristic?
- Why do we use meta-heuristic methods like Evolutionary Algorithms instead of classical, analytical methods?
- What is the difference between a single- and a multi-objective optimization problem?
- Explain the concept of Pareto-dominance and why it is used in multi-objective optimization.
- Consider the car example from the lecture. You want to buy a new car and have three criteria: Price (which should be minimized), gas consumption (which should me minimized) and maximum speed (which should be maximized). The cars and their properties are listed as follows.

	Price (EUR)	Fuel consumption	Max speed (km/h)
VW	16,200	7.2	180
Opel	14,900	7.0	220
Ford	14,000	7.5	200
Toyota	15,200	8.2	250

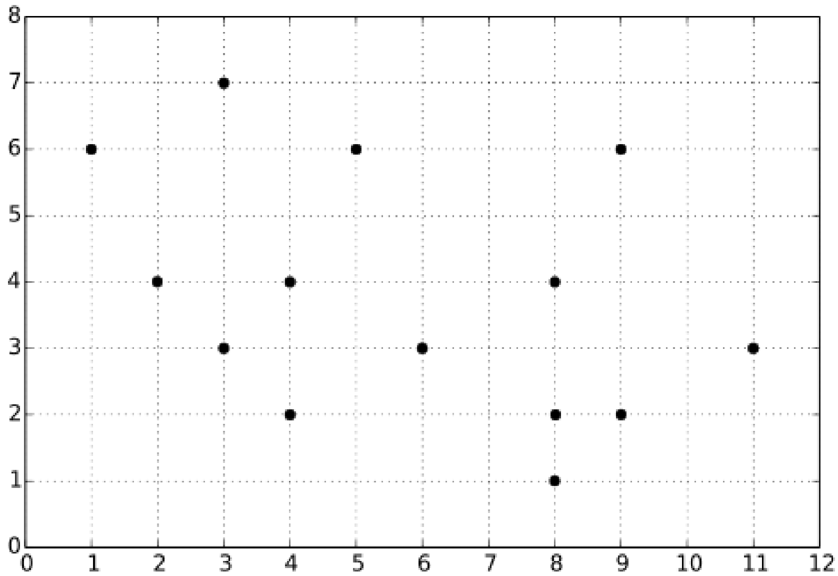
  Using Pareto-dominance, which of these cars are non-dominated? Which of these cars would you buy as a decision maker and why?

Assignment 2 **Pareto-Dominance Concepts**

Please answer the following questions regarding Pareto-dominance.

- What are the ideal points and nadir points in the context of multi-objective optimization?
- The following picture shows a set of solutions in a 2-dimensional objective space. ($f_1(\vec{x})$ on the x-axis and $f_2(\vec{x})$ on the y-axis). Indicate which of these solutions are non-dominanted with regard to Pareto-optimality if

  a) f_1 is minimized and f_2 is minimized
  b) f_1 is minimized and f_2 is maximized
  c) f_1 is maximized and f_2 is minimized
  d) f_1 is maximized and f_2 is maximized



Are the resultant non-dominated sets convex or concave? What are the ideal and nadir points of these sets (assuming there are no other solutions in the search space)? Indicate which solutions are non-dominated with regard to weak Pareto-optimality.

Assignment 3

We are going this holiday on a trip to Munich, and we are looking for a good hotel with the best quality (defined by the highest number of stars of the hotel), the lowest price, the lowest distance to the city center and the highest number of beds. A set of possible hotels is listed as follows.

	Quality ($f_1(x)$)	Price ($f_2(x)$)	Centre Distance ($f_3(x)$)	Beds ($f_4(x)$)
Hotel Mozart (x_1)	3 Stars	110EURO/night	6km	400 beds
Hotel Verdi (x_2)	2 Stars	120EURO/night	2km	250 beds
Hotel Chopin (x_3)	1 Star	70EURO/night	4.5km	600 beds
Hotel Vivaldi (x_4)	2 Stars	105EURO/night	1km	200 beds
Hotel Beethoven (x_5)	3 Stars	124EURO/night	6km	350 beds
Hotel Bach (x_6)	1 Star	71EURO/night	3km	600 beds
Hotel Wagner (x_7)	5 Stars	170EURO/night	3km	100 beds
Hotel Hendel (x_8)	3 Stars	124EURO/night	6km	350 beds
Hotel Brahms (x_9)	3 Stars	124EURO/night	6km	250 beds
Hotel Schubert (x_{10})	2 Stars	120EURO/night	1km	500 beds

- Using Pareto-dominance, which of these hotels are non-dominated?
- If we normalize each objective, so all the values lie in the interval $[0, 1]$, what solutions will be non-dominated?

Assignment 4 Distance Minimization Problem (DMP)

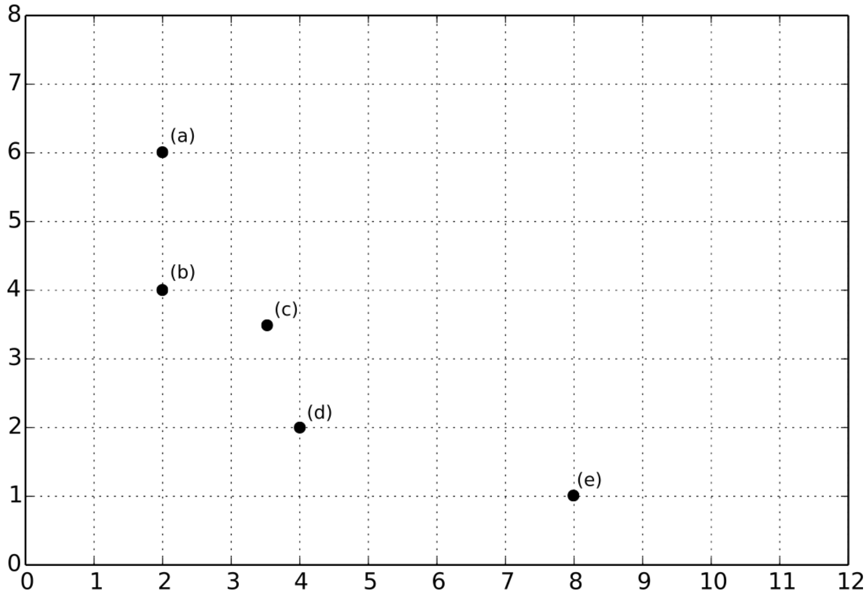
Assume you just moved to a new town and want to find a place to live for the next 3 years. However, you care about being close to certain locations in town in order to get the best living quality. It is very important for you to live close to the university and the supermarket.

- In order to be able to make a good choice for your new flat's location, you want to formulate this problem as a multi-objective optimization problem (explain which are the decision variables and the objective variables). Please describe how this can be done and explain where the optimal locations lie on a small example.
- In addition, you also want to be close to the gym. With these three points, what would be the new multi-objective optimization problem? Please explain how the Pareto-optimal solutions change compared to the previous case with only two points of interest.

Assignment 5 A-Priori Methods

In the lecture we have learned about different methods to solve optimization problems. In the following, we pay attention to a priori methods.

- Explain the difference between a priori methods, interactive methods and a posteriori methods.
- Explain how the Weighting Method from the lecture works. Which advantages and disadvantages does this have? In the following set of solutions, where both objectives are minimized, which solution(s) are optimal if we use the Weighting methods with the weights $(w_1, w_2) := (0.5, 0.5)$? Which ones are optimal for the weights $(0.1, 0.9)$?



- Explain how the ε-Constraint Method from the lecture works. Which advantages and disadvantages does this have? Which solution(s) are optimal if we optimize f_1 and set a constraint for f_2 of $\varepsilon_2 := 5.0$? How does this change if we change the constraint to $\varepsilon_2 := 3.0$?
- Explain how the lexicographic ordering of objectives from the lecture works. Which advantages and disadvantages does this have? In the above example, wich solution(s) are optimal if we specify the importance of objectives as $f_1 >> f_2$? Which solution(s) are optimal if we reverse the order?

Assignment 6 A Priori Methods in Higher Dimensions

Looking back to our holiday trip to Munich (Assighment 3), we would now like to apply some a priori method to help us pick a good hotel with the best quality (defined by the highest number of stars of the hotel), the lowest price, the lowest distance to the city centre and the highest number of beds. For this, we have normalized the features of each hotel (using min-max normalization). The set of possible hotels is listed as follows.

	f_1 Normalized Quality (to be maximized)	f_2 Normalized Price (to be minimized)	f_3 Normalized Distance (to be minimized)	f_4 Normalized Beds (to be maximized)
x_1	0.5	0.4	1	0.6
x_2	0.25	0.5	0.2	0.3
x_3	0	0	0.7	1
x_4	0.25	0.35	0	0.2
x_5	0.5	0.54	1	0.5
x_6	0	0.01	0.4	1
x_7	1	1	0.4	0
x_8	1	1	0.4	0.5
x_9	1	1	0.4	0.3
x_{10}	1	1	0.4	0.8

- Using the a priori weighting method on the normalized problem, with a weight of 0.2 for quality, a weight of 0.5 for price, a weight of 0.2 for distance to the city centre and a weight of 0.1 for number of beds; which of the hotels are optimal?
- Using the a priori ϵ-constraint method, which hotels are optimal if we set $\epsilon_1 = 0.5$ and $\epsilon_3 = 0.5$?

Assignment 7 Cone-Domination

Revise the concept of cone domination from the lecture and answer the following questions.

- What is the concept of cone domination and how does it work?
- Using the solutions shown in Assignment 5, which of the points are non-cone-dominated using the following matrix
$$\begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}$$
How does this compare to normal Pareto-dominance?
- Using the solutions shown in Assignment 5, which solution are non-cone-dominated with the following matrix
$$\begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$$
- Which matrix has to be used in cone domination if we want to dominate solutions with a symmetric angle of $\varphi = 160$ degrees.

Assignment 8 ϵ-Domination

Revise the concept of ϵ-domination from the lecture and answer the following questions.

- What is ϵ-domination and how does it work?
- Using the solutions shown in Assignment 5, which of the points are non-ϵ-dominated using $\epsilon = 0.5$?
  Note: Use the first definition given, not the simplified version.
- Using the solutions shown in Assignment 5, which of the points are non-ϵ-dominated using $\epsilon = 1$? What is the smallest ϵ-approximated Pareto front in this case?
  Note: An ϵ-approximated Pareto front PF_ϵ consists of a set of solutions where for any solution u in the problem, there exists a solution $v \in PF_\epsilon$ such that v ϵ-dominates u.
- What ϵ value can be used in order to assure that only solutions b and d are non-ϵ-dominated?

Assignment 9 Problem Formulation

Assume you work in a car manufacturing company, and your task is to design the structure of a new car. Your goal is to minimize the wind resistance of the car, while at the same time using only a fixed amount of material. The 3-D model of the car can be described by 7 positive real-valued parameters. Because there is the restriction of using at most a fixed amount of material, the sum of these parameters cannot be higher than 100. To test the wind resistance of a concrete assignment of these parameters, the values first have to be transformed into a 3-D model of a car. Then, using a simulation environment, the wind resistance can be determined.

- You have decided to use an Evolutionary Algorithm to solve this optimization problem. Please indicate which parts in the above description correspond to (1) the search space S, (2) the feasible search space F, (3) the decoded search space W, (4) the solution space G, (5) the decoding function d and (6) the cost function g of this problem.
- Describe a suitable representation of solutions and the optimization problem mathematically. What is the fitness function?
- What could be a possible neighborhood function that does not leave the feasible space F when starting from a solution in F.

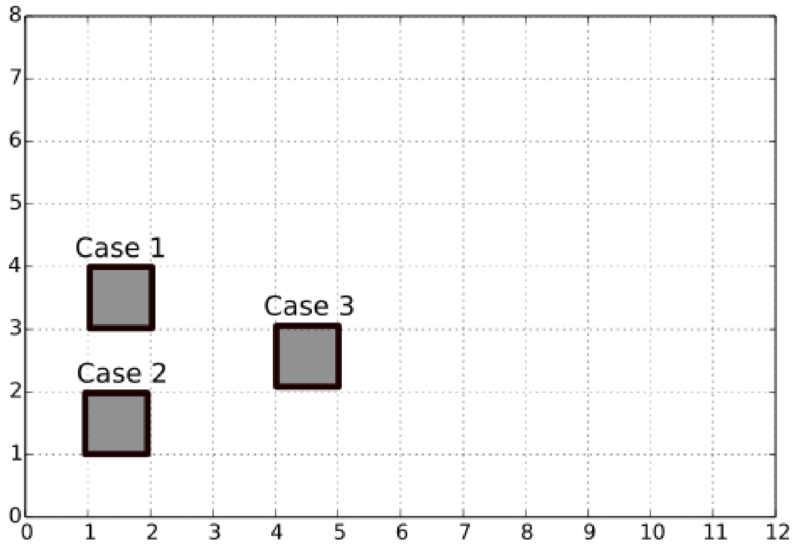
Assignment 10 Interactive Methods

Review the information about interactive methods and answer the following questions.

- Present three possible forms of interaction with the decision maker (DM).
- What characteristics should a good interactive method possess with respect to the interaction with the DM?
- We want to solve the following bi-objective optimization problem:

$$\min(x_1 + 2x_2 + 3x_3, -2x_1 - 3x_3 + 6)$$
$$s.t. x_1, x_2, x_3 \in \{0, 1\}$$

For which we use a binary encoding with three bits, and a standard deterministic 1-bit neighborhood. In a first iteration, the solutions $\vec{a} = (1, 0, 0)$, $\vec{b} = (0, 1, 0)$, $\vec{c} = (0, 0, 1)$, $\vec{d} = (0, 1, 1)$ and $\vec{e} = (1, 0, 1)$ were obtained.
Then, only the two most relevant between these solutions should be presented to the DM. For deciding which ones, the DM provides a rectangular region in the objective space that he/she considers of higher interest. What solutions would you present in each of the following cases?



- For the same problem, each of the five solutions are updated by taking a solution in their neighborhood or keeping the current solution. This is done based on the region provided by the DM, so the selected solution is the best possible according to the DM preferences. Which new five solutions would you obtain for each of the previous cases?
Note: When selecting the best solution in the neighborhood, a non-dominated solution (in the neighborhood) is always preferred, and the closeness to the region is used as a second factor.

Assignment 11 Hill-Climbing

Make yourself familiar with the Hill Climbing Algorithm as introduced in the lecture.

- What are the advantages and disadvantages of using the Hill Climbing algorithm as optimization algorithm?
- Make yourself familiar with the Traveling Salesman Problem (TSP) as introduced in the lecture. Your next task is to apply the Hill Climbing algorithm to the TSP. The distance matrix mat_{dist} indicates the distances between the cities c_1 to c_5:

$$\mathrm{mat}_{dist} = \begin{bmatrix} 0 & 110 & 350 & 220 & 70 \\ 110 & 0 & 455 & 260 & 170 \\ 350 & 455 & 0 & 420 & 490 \\ 220 & 260 & 420 & 0 & 170 \\ 70 & 170 & 490 & 170 & 0 \end{bmatrix}$$

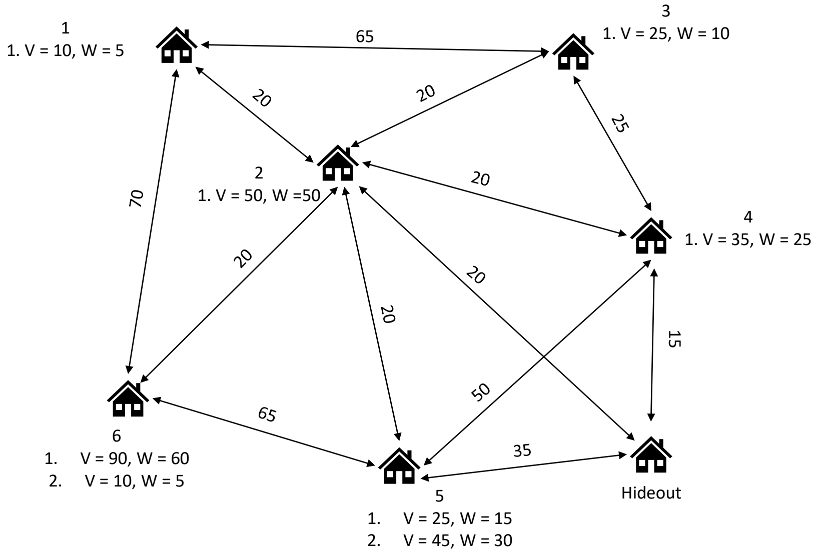
  where c_1: Los Angeles, c_2: San Diego, c_3: San Francisco, c_4: Las Vegas, c_5: California City.

  The Salesman does not need to come back to the city where he started, i.e. after visiting all five cities he will stay in the last city.

  Use the permutation representation of the TSP and exchange two cities as a neighborhood function. The fitness function of the problem is the sum of all distances, which should be minimized. Start with c_2-c_4-c_1-c_5-c_3 as initial solution x_c and apply the algorithm until the stopping criterion is fulfilled.

Assignment 12 Traveling Thief Problem

Assume you are part of the most famous thieves guild in Germany. You and your fellow thieves have come up with a plan to rob several locations in one night and retire. Each location has some valuable items. You know the value of the items along with their weight in kilograms. You have created a graph were the nodes correspond to the locations and the edges represent the distances between locations:



Everyone in the guild was assigned a task and you are responsible for planing the operation, while keeping certain things in mind:

- You must start from the Hideout and return to the Hideout after the operation.
- The team executing the operation can only carry a maximum weight of $Q = 200[kg]$
- The team start out with a maximum speed $V_{max} = 1$
- The team speed is decreased as the weight they carry increases $V = V_{max} - q/Q$ ($q \equiv$ the current weight the team is carrying)
- The time taken to go from one location to another is $t = V * distance$

Your goal is to maximize your profit and minimize the time it takes to execute the operation so the police has a lower chance of catching you. How can you model and formulate this problem? Which encoding and neighborhood function would you choose? Explain your reasoning!