Revision of Probabilistic Graphical Models

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Graphical models are efficient for representing domain knowledge. After some time additional observations can change our underlying knowledge of the domain.

We need a way to incorporate those changes to avoid updating the whole knowledge base.

Idea: local changes should only lead to local adaptations of the knowledge base and neccessary consequences.

Example: It is known that a certain navigation system can only be included in the car if one of the corresponding radio systems is already installed. It is planned to sell 3000 instead of 1000 navigation systems in the next quartal. How many radio systems of each type should be bought.

Revision of Probabilistic Graphical Models

Prior Probability Distribution

New dimensional conditional probabilities

Revision

Principle of minimal Change

Posterior Probability Distribution including specified **and** inferred changes

Information-theoretically closest to the prior distribution

Revision Operator - Iterative Proportional Fitting

Iterative proportional fitting is a well-known algorithm for adapting the marginal distributions of a given joint distribution to desired values.

It consists in computing the following sequence of probability distributions:

$$
\begin{aligned}
p_U^{(0)}(u) &\equiv p_U(u) && (1)\\
\forall 1, 2, \cdots : p_U^{(i)}(u) &\equiv p_U^{(i-1)}(u) \frac{p_{A_j}(a)}{p_{A_j}^{(i-1)}(a)} && (2)
\end{aligned}
$$

In each step the probability distribution is modified in such a way that the resulting distribution satisfies the given marginal distribution A_j. However, this will, in general, change the marginal distribution for an earlier adapted variable A_k.

Therefore, the adaptation has to be iterated, traversing the set of variables several times. The process is proofed to converge for non-contradicting revision statements.

The revision algorithm sums up as follows:

forall $C \in \mathcal{C}$ **do**
  $p_C^{(0)}(c) \equiv p_C(c)$
$i \equiv 0$
repeat
$i \equiv i + 1;$
  forall $C \in \mathcal{C}$ **do**
    forall $j \in J_C$ **do**
      $p_C^{(i)}(c) \equiv p_C^{(i-1)}(c) \dfrac{p_{A_j}(a)}{p_{A_j}^{(i-1)}(a)};$
    do evidence propagation
  end
until convergence

Inconsistencies

Inconsistencies can emerge in the presence of:

Complex structure: dependencies between attributes
e.g. dependencies between car components

Many revision assignments: changes of the probability distribution
e.g. changing installation rate of component combinations

Inconsistencies are unincorporatable changes / inconsistent revision assignments

Inner Inconsistencies

Revision assignments are inconsistent independent of prior distribution

Outer Inconsistencies

Revision assignments inconsistent with zero-values in prior distribution

Example: Inner Inconsistencies

Insert revision assignments in probability distribution:

0.6	0.3		
0.2	0.25		
0.3			0.5
		0.1	

Inner inconsistencies can emerge as consequences of probability implications:

0.6	0.3	0.1	$\leftarrow 0.10 = 1 - 0.6 - 0.3$
0.2	0.25		
0.3			0.5
0.1		0.1	$\leftarrow 0.10 = 0.6 - 0.2 - 0.3$

Example: Inner Inconsistencies

Insert revision assignments in probability distribution:

0.6	0.3		
0.2	0.25		
0.3			0.5
		0.1	

Inner inconsistencies can emerge as consequences of probability implications:

0.6	0.3	0.1		
0.2	0.25	0.0		← set to zero since column sum is already maximum
0.3		0.0	0.5	← set to zero since column sum is already maximum
0.1		0.1		

Example: Inner Inconsistencies

Insert revision assignments in probability distribution:

0.6	0.3		
0.2	0.25		
0.3			0.5
		0.1	

Inner inconsistencies can emerge as consequences of probability implications:

0.6	0.3	0.1		
0.2	0.25	0.0	0.45	$\leftarrow 0.45 = 0.2 + 0.25 + 0.0$
0.3		0.0	0.5	
0.1		0.1	0.05	

Example: Inner Inconsistencies

Insert revision assignments in probability distribution:

0.6	0.3		
0.2	0.25		
0.3			0.5
		0.1	

Inner inconsistencies can emerge as consequences of probability implications:

0.6	0.3	0.1	
0.2	0.25	0.0	0.45
0.3		0.0	0.5
0.1		0.1	?

Contradicting implications: $0.05 \neq 0.20$

column-sum $\Rightarrow 1 - 0.45 - 0.5 = 0.05$

row-sum $\Rightarrow 0.1 + 0.1 = 0.20$

Example: Outer Inconsistencies

Insert revision assignments in probability distribution with fixed zero values ($\times$):

0.1	0.5	0.4	
	$\times$	$\times$	0.3
	$\times$		0.5

Outer inconsistencies can emerge as consequences of probability implications:

0.1	0.5	0.4	
	$\times$	$\times$	0.3
	$\times$		0.5
	0.5		0.2

Contradicting implications: $0.5 > 0.2$

column-sum $\Rightarrow 0.5 - 0.0 - 0.0 = 0.5$

column-sum $\Rightarrow 1.0 - 0.3 - 0.5 = 0.2$

Example: Outer Inconsistencies

Even for an expert user it is not easy to configure revision statements without creating inconsistencies!

If the Revision-Operation fails, we need to explain the user how to change his desired revision statements. Otherwise no solution can be found.

Systematic Handling of Inconsistencies

1. Detection

Heuristical Methods

2. Automatic Resolution

Partition Mirrors

3. Analysis

Finding Minimal Explaining Set
Grouping Assignments

4. Explanation

Displaying minimal explaining set and structure

(Handling Revision Inconsistencies:
Creating Useful Explanations, *Schmidt, F.; Gebhardt, J.; Kruse, R.*)