Bayes Networks Winter 2018/2019

Prof. Dr. R. Kruse / Alexander Dockhorn / Alishiba Dsouza

Exercise Sheet 10

Markov Properties of Undirected Graphs

Let $(\cdot \perp\!\!\!\perp \cdot \mid \cdot)$ be the ternary relation that represents the conditional independence statements that hold true in a probability distribution p over a common domain and set V of attributes. An undirected graph $G = (V, E)$ satisfies the

pairwise Markov property
if and only if every pair of non-adjacent attributes in the graph are conditional independent in p given all other attributes,i. e.

$$\forall A, B \in V, A \neq B : (A, B) \notin E \Rightarrow A \perp\!\!\!\perp B \mid V \backslash \{A, B\}.$$

G has the **local Markov property**
if and only if every attribute in p is conditionally independent of all others given its neighbors,i. e.

$$\forall A \in V : A \perp\!\!\!\perp V \backslash \{A\} \backslash \text{neighbors}(A) \mid \text{neighbors}(A),$$

with $\text{neighbors}(A) = \{B \in V \mid (A, B) \in E\}$,

G has the **global Markov property**
if and only if from u-separation of two sets of attributes given a third one it follows that these two sets are conditionally independent in p given the third one, i. e.

$$\forall X, Y, Z \subseteq V : \langle X \mid Z \mid Y \rangle_G \Rightarrow X \perp\!\!\!\perp Y \mid Z.$$

Exercise 31 Markov Properties of Undirected Graphs

Consider the following graph:

$$\text{A} - \text{B} - \text{C} - \text{D} - \text{E}$$

Let $\text{dom}(A) = \cdots = \text{dom}(E) = \{0, 1\}$. Assuming the probability distribution $P(A = 0) = P(E = 0) = \frac{1}{2}$, $A = B$ (i. e. $P(B = 0 \mid A = 0) = 1$ and $P(B = 1 \mid A = 1) = 1$), $D = E$ and $C = B \cdot D$, show that the graph satisfies the pairwise and local but not the global Markov property.

Exercise 32 Exam Preparation

The exam is rapidly approaching. Prepare yourself for the following topics so we can actively discuss (some of them were not part of the exercise classes):

- differences of vagueness, imprecision, and unreliabity
- basic probability axioms
- conditional and unconditional probabilities, Bayes Theorem
- conditional, unconditional, pairwise and full independence
- decomposition of relations / probability distributions
- projection/marginalization, cylindrical extension
- u/d-separation
- bayesian networks
- (semi-)graphoid axioms
- simple tree propagation
- clique tree construction
- clique tree propagation
- parameter learning
- Naive Bayes Classifier
- likelihood of a database
- structure learning
- decision trees
- K2 algorithm
- Revision
- **...all topics after revision will not be relevant for the exam**