Exercise Sheet 9

Exercise 28 Learning from Data

Assume the following conditional independencies between the four attributes A, B, C and D (as in former exercises, the notation $X \perp\!\!\!\perp Y \mid Z$ states that X is independent of Y given Z):

$$A \perp\!\!\!\perp B \mid \emptyset, \qquad A \perp\!\!\!\perp D \mid C, \qquad B \perp\!\!\!\perp D \mid C$$

Assume further that only these independencies as well as those that are deducible by the graphoid axioms (cf. lecture slides) hold true (i.e. the symmetric conuterparts $B \perp\!\!\!\perp A \mid \emptyset$ etc. are satisfied). All other conditional independencies do not hold true. Which conditional independence graph over the four attributes can be read from this information?
(Hint: Remember the special properties of converging edges.)

Exercise 29 Learning from Data

A simple approach to learn a graphical model from data consists in constructing an optimal spanning tree w.r.t. edge weights that represent the strengths of the attributes connected by that edge. Such a tree is named after its inventors Chow-Liu tree. We consider here the construction of a maximal spanning tree in the relational setting with the Hartley information gain

$$\begin{aligned} I_{\text{gain}}^{(\text{Hartley})}(A,B) &= \log_2\left(\sum_{i=1}^{n_A} R(A=a_i)\right) + \log_2\left(\sum_{j=1}^{n_B} R(B=b_j)\right) \\ &- \log_2\left(\sum_{i=1}^{n_A}\sum_{j=1}^{n_B} R(A=a_i, B=b_j)\right) \\ &= \log_2 \frac{\left(\sum_{i=1}^{n_A} R(A=a_i)\right)\left(\sum_{j=1}^{n_B} R(B=b_j)\right)}{\sum_{i=1}^{n_A}\sum_{j=1}^{n_B} R(A=a_i, B=b_j)}. \end{aligned}$$

as the measure to assess the strength of dependence between attributes A and B: Determine for the relation from exercise 13 (repeated below) the Chow-Liu tree w.r.t. the Hartley information gain! Compare the result with the result of exercise 13!

A	a_1	a_1	a_2	a_2	a_2	a_2	a_3	a_3
B	b_1	b_1	b_1	b_1	b_3	b_3	b_1	b_2
C	c_1	c_2	c_2	c_3	c_2	c_3	c_2	c_2

Exercise 30 Learning from Data

Consider the following probability distribution:

	$C = c_1$		$C = c_2$	
	$B = b_1$	$B = b_2$	$B = b_1$	$B = b_2$
$A = a_1$	$4/35$	$12/35$	$4/35$	$1/35$
$A = a_2$	$1/35$	$3/35$	$8/35$	$2/35$

Determine the Chow-Liu tree for that distribution w.r.t. the Shannon information gain

$$I_{\text{gain}}^{(\text{Shannon})}(A, B) = \sum_{i=1}^{n_A} \sum_{j=1}^{n_B} P(A = a_i, B = b_j) \log_2 \frac{P(A = a_i, B = b_j)}{P(A = a_i) \cdot P(B = b_j)},$$

i.e. use the Shannon information gain as the edge weight and determine the maximal spanning tree! Does the result represent a correct decomposition?