Exercise Sheet 4

Exercise 13 Decompositions of Relations

Consider the given relation over the three attributes A, B and C, whose domains contain three values each, i.e., $\text{dom}(A) = \{a_1, a_2, a_3\}$, $\text{dom}(B) = \{b_1, b_2, b_3\}$ and $\text{dom}(C) = \{c_1, c_2, c_3\}$. How can this relation be decomposed into projections onto subspaces? How can the original relation be reconstructed from these projections? Which (single) Tuples can be removed from the original relation without violating the decomposability? Why is it not possible to remove the other tuples? Which (single) tuples can be added to the original relation without violating the decomposability? Why can no other tuple be added?

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_2	b_1	c_2
a_2	b_1	c_3
a_2	b_3	c_2
a_2	b_3	c_3
a_3	b_1	c_2
a_3	b_2	c_2

Exercise 14 Conditional relational Independence

The relation from exercise 13 can be decomposed into projections because a conditional independence holds true. Which? How can this conditional independence be verified? (Hint: Remember that projections of relations can be represented by a maximum operation on the indicator function of the relation. Intersections of the cylindrical extensions can be modelled by a minimum operation. The indicator function of a relation returns value 1 for every tuple contained in the relation and returns value 0 for all others.)

Exercise 15 Conditional Independence

Which conditional and unconditional (marginal) dependencies and independencies hold true between the following variables:

a) Position of accelerartor pedal, engine rotation speed, vehicle speed
b) Amount of precipitation, amount of deployed fertilizer, crop yield
c) Number of swimming accidents, amount of consumed ice cream, outside temperature
d) Number of storks, birth rate, coffee price

What do the respective graph structures look like?

Additional Exercise Constructing Bayesian Networks

Construct the graph of a Bayesian network that models the following situation [Finn V. Jensen: An Introduction to Bayesian Networks, UCL Press, London, UK 1996]:

> Mr Holmes is working at his office when he receives a telephone call from Watson, who tells him that Holmes burglar alarm has gone off. Convinced that a burglar has broken into his house, Holmes rushes to his car and heads for home. On the way he listens to the radio, and in the news it is reported that there has been a small earthquake in the area. Knowing that earthquakes have a tendency to turn the burglar alarm on, he returns to his work leaving his neighbours the pleasure of the noise.

Of course, there a various ways of modelling this scenario. Please keep you model simple and introduce only variables that are necessary for Mr Holmes to assess the situation! Specify the factorization that is implied by your graph! Finally, determine the probability distributions from the following facts:

- Burglary is rare, earthquakes even rarer.
- There are very little earthquakes about which there is no news report.
- A false report about an earthquake is extremely rare.
- The alarm is quite reliable, however, during an earthquake it might easily produce a false alarm.
- A false alarm with another cause is possible, albeit unlikely.

The phrases, of course, do not fix any numbers. Choose meaningful values that model the linguistic propositions adequately!