Inference in Belief Trees

Inference in Belief Networks

A Bayesian Network is a complete model for the variables and their relationships.

It can be used to answer queries about them.

Typical question:
Given observes variables, what is the updated knowledge about the other variables.

There are exact inference methods for this task.

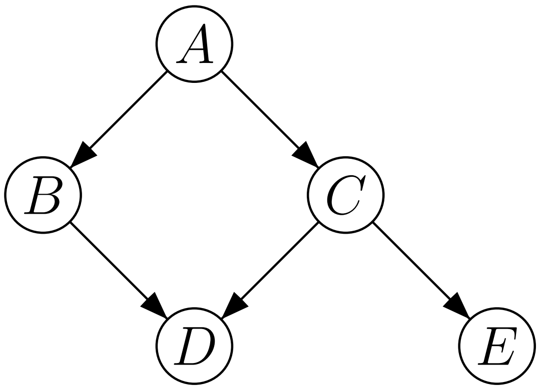
Choice of universe of discourse

	Variable	Domain
A	metastatic cancer	$\{a_1, a_2\}$
B	increased serum calcium	$\{b_1, b_2\}$
C	brain tumor	$\{c_1, c_2\}$
D	coma	$\{d_1, d_2\}$
E	headache	$\{e_1, e_2\}$

($\cdot_1$ — present, $\cdot_2$ — absent)

$$\Omega = \{a_1, a_2\} \times \cdots \times \{e_1, e_2\}$$
$$|\Omega| = 32$$

Analysis of dependencies



Motivation

$$\left.\begin{aligned} P(e_1 \mid c_1) &= 0.8 \\ P(e_1 \mid c_2) &= 0.6 \end{aligned}\right\} \text{headaches common, but more common if tumor present}$$

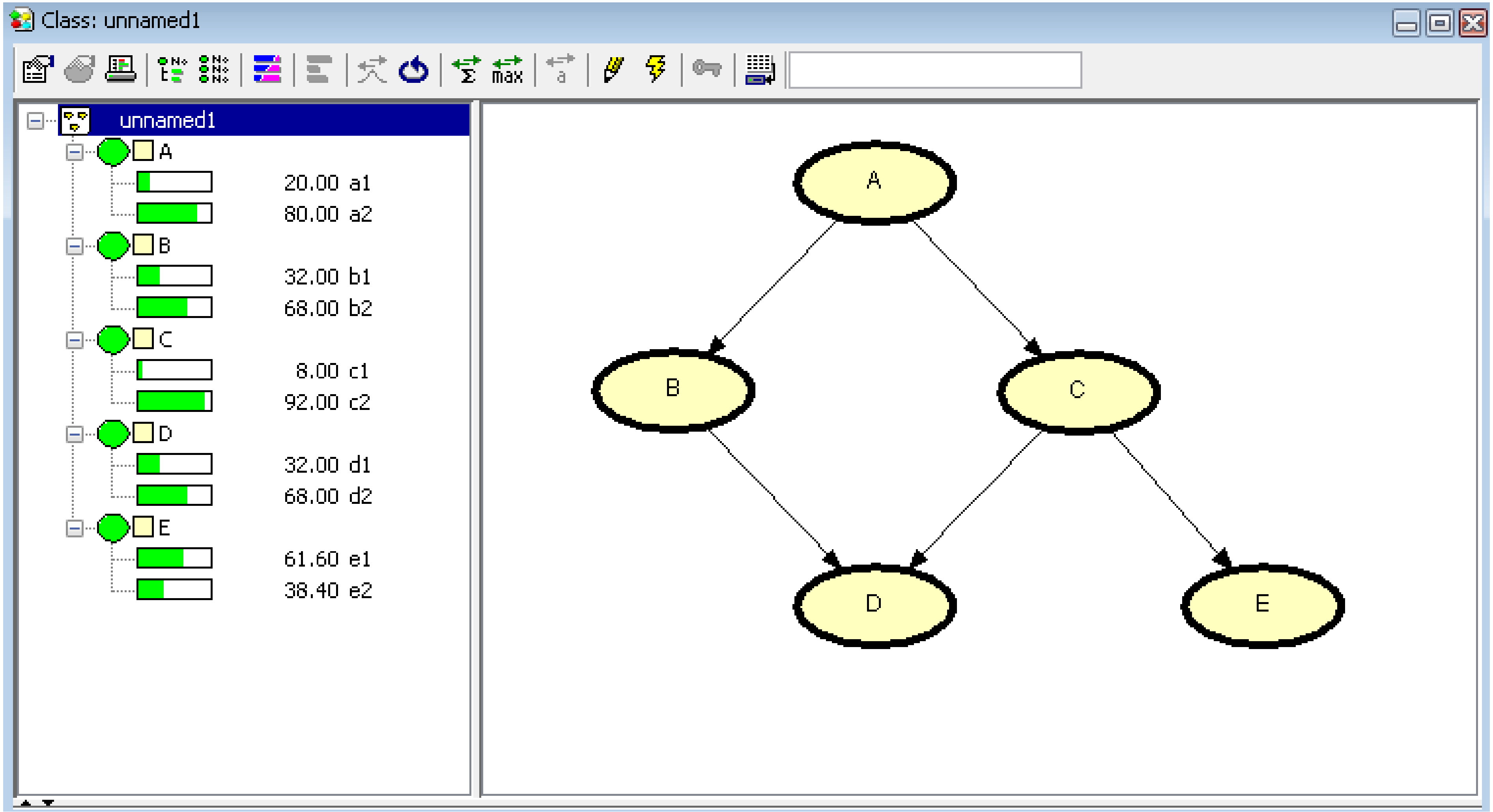
$$\left.\begin{aligned} P(d_1 \mid b_1, c_1) &= 0.8 \\ P(d_1 \mid b_1, c_2) &= 0.8 \\ P(d_1 \mid b_2, c_1) &= 0.8 \\ P(d_1 \mid b_2, c_2) &= 0.05 \end{aligned}\right\} \text{coma rare but common, if either cause is present}$$

$$\left.\begin{aligned} P(b_1 \mid a_1) &= 0.8 \\ P(b_1 \mid a_2) &= 0.2 \end{aligned}\right\} \begin{array}{c}\text{increased calcium uncommon,} \\ \text{but common consequence of metastases}\end{array}$$

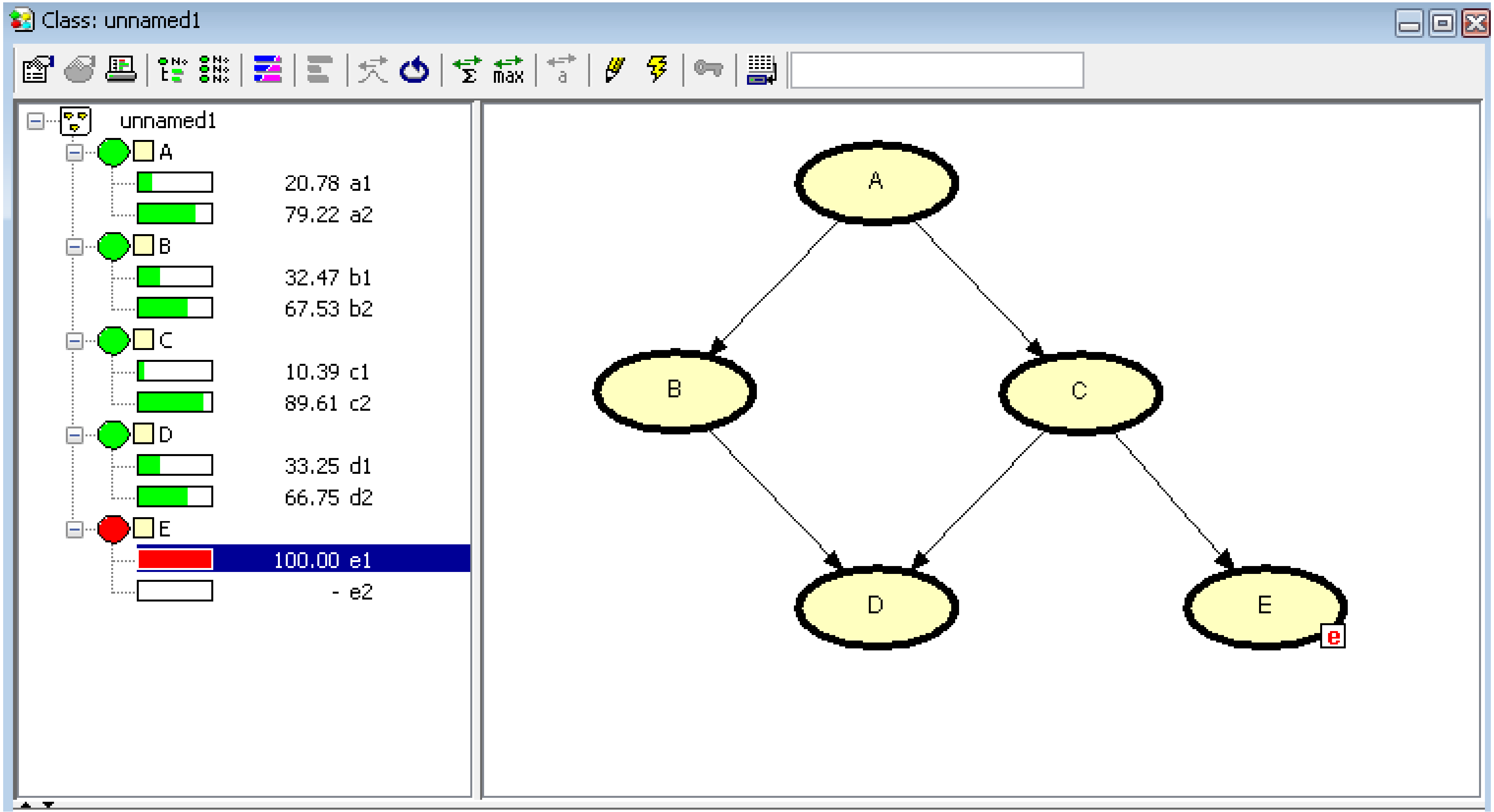
$$\left.\begin{aligned} P(c_1 \mid a_1) &= 0.2 \\ P(c_1 \mid a_2) &= 0.05 \end{aligned}\right\} \text{brain tumor rare, and uncommon consequence of metastases}$$

$$P(a_1) = 0.2 \quad \} \ \text{incidence of metastatic cancer in relevant clinic}$$

Motivation

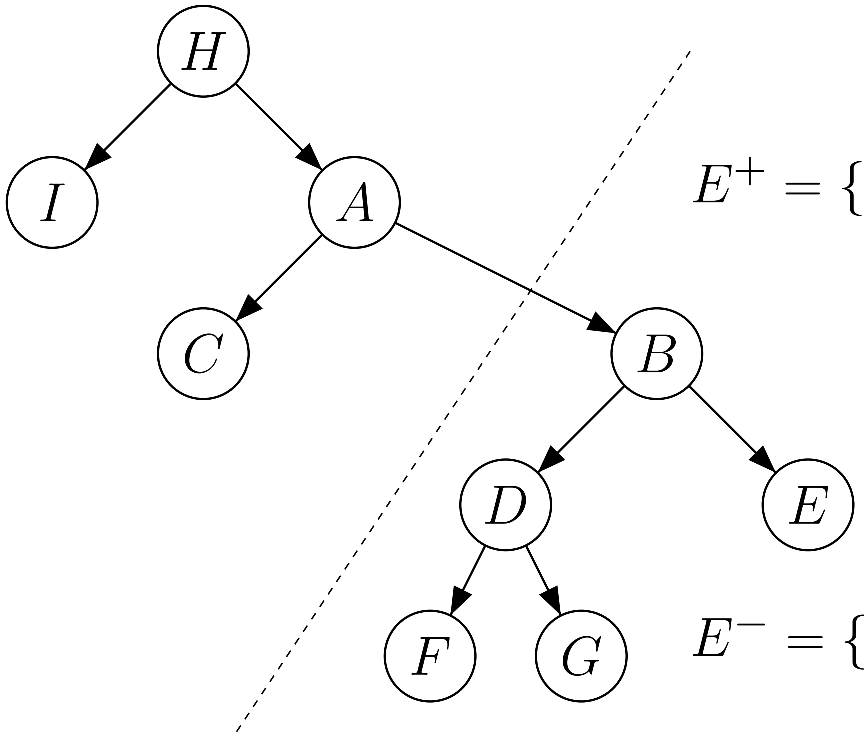


Marginal distributions in the HUGIN tool.



Conditional marginal distributions with evidence $E = e_1$

Given that the BN has a tree structure, any node divides the network into two independent subtrees



$E^{+} = \{H, A, I, C\}$

$E^{-} = \{B, D, E, F, G\}$

Basic Equations

Given certain evidence, E (subset of instantiated variables), the posterior probability for a value i of any variable B, can be obtained by applying the Bayes rule:

$$P(B_i|E) = P(B_i)P(E|B_i)/P(E)$$

We can separate the evidence into:

- E^-: Evidence in the tree rooted in B.
- E^+: All other evidence.

Then:

$$P(B_i|E) = P(B_i)P(E^-, E^+|B_i)/P(E)$$

Basic Equations

Given that E^+ and E^- are independent, by applying the Bayes rule again, we obtain.

$$P(B_i|E) = \alpha P(B_i|E^+)P(E^-|B_i)$$

Where α is a normalization constant.

We define the following terms:

$$\lambda(B_i) = P(E^-|B_i)$$

$$\pi(B_i) = P(B_i|E^+)$$

Then:

$$P(B_i|E) = \alpha\pi(B_i)\lambda(B_i)$$

Propagation Algorithm

The computation of the posterior probability of any node B is decomposed into two parts:

- the evidence coming from the sons of B in the tree (λ)
- and the evidence coming from the parent of B, (π)

Each node B in the tree is like a simple processor that stores

- its vectors $\pi(B)$ and $\lambda(B)$
- its conditional probability table, $P(B|A)$

Evidence is propagated via a message passing mechanism

- each node sends the corresponding messages to its parent and sons in the tree

Messages

A message sent from node B to its parent A

$$\lambda_B(A_i) = \sum_j P(B_j|A_i)\lambda(B_j)$$

A message sent from node B to its son S_k :

$$\pi_k(B_i) = \alpha\pi(B_j)\prod_{l\neq k}\lambda_l(B_j)$$

where l refers to each one of the sons of B

Combination and Propagation

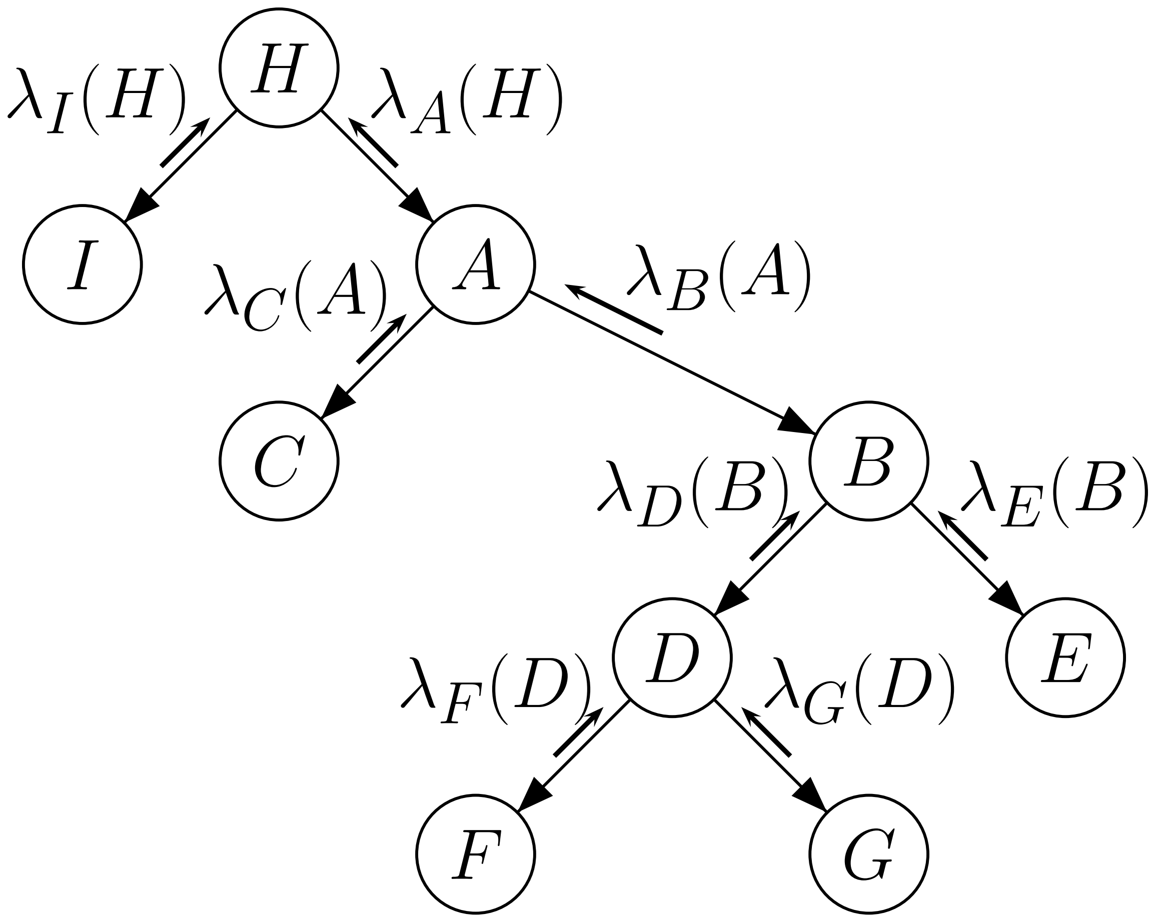
Each node can receive several λ messages, which are combined via a term by term multiplication for the λ messages received from each son (S_k):

$$\lambda(B_i) = \prod_k \lambda_{S_k}(B_i)$$

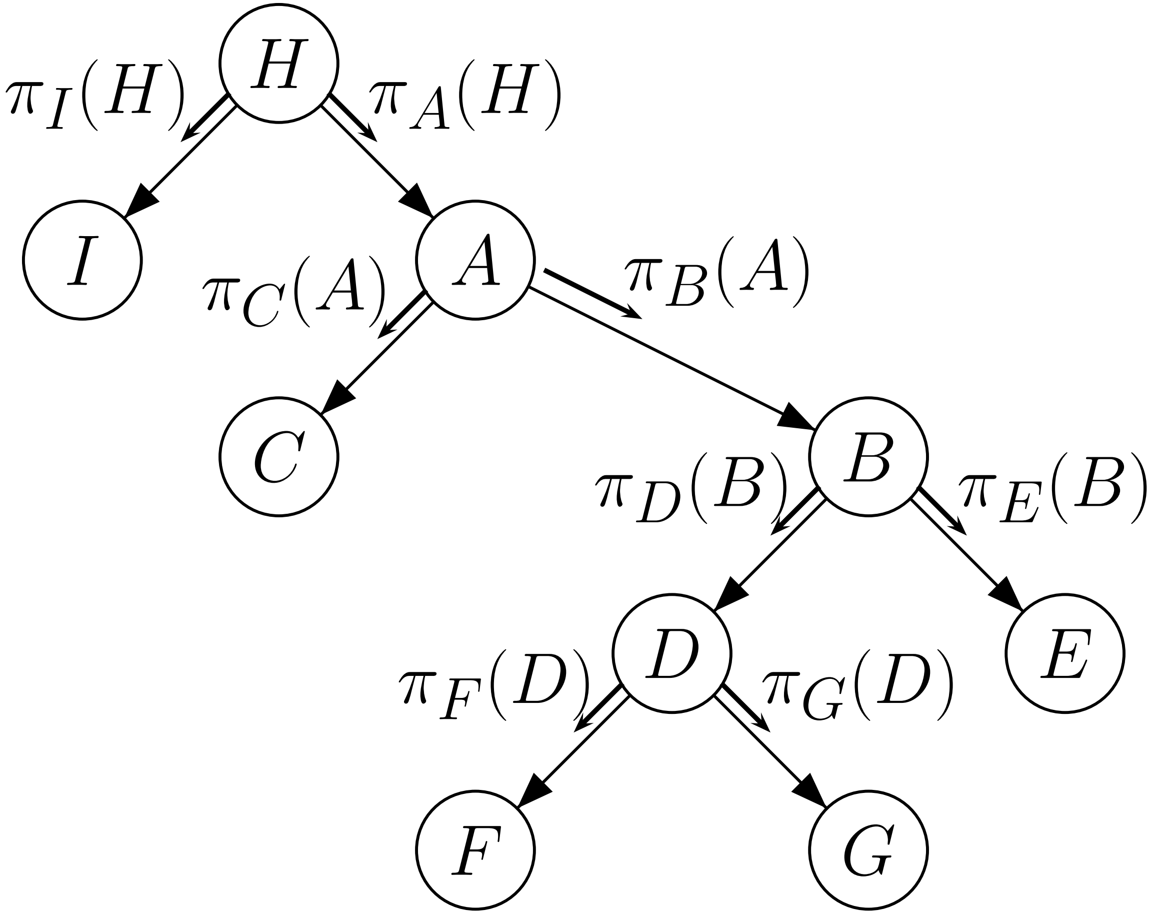
The propagation algorithm starts by assigning the evidence to the known variables, and then propagating it through the message passing mechanism until:

- the root of the tree is reached for the λ messages
- the leaves are reached for the π messages

Bottom-up propagation



Top-down propagation



Initial Conditions

Leaf nodes:

- If not known, $\lambda = [1, 1, \ldots, 1]$ (a uniform distribution)
- If known, $\lambda = [0, 0, \ldots, 1, \ldots, 0]$ (one for the assigned value and zero for all other values).

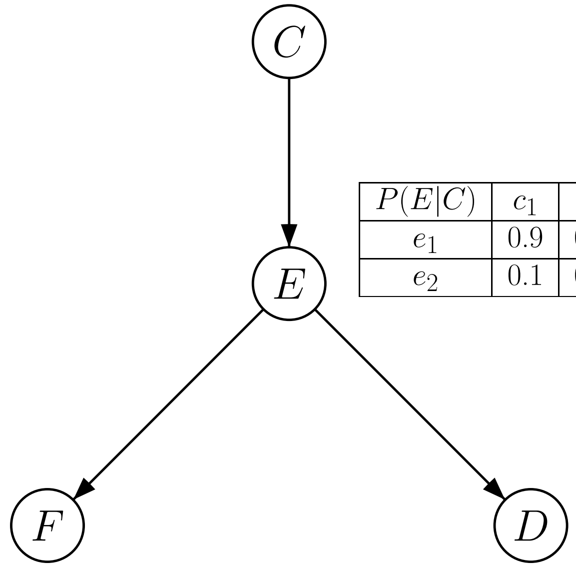
.

Root node:

- If not known, $\pi = P(A)$ (prior marginal probability vector)
- If known, $\pi = [0, 0, \ldots, 1, \ldots, 0]$ (one for the assigned value and zero for all other values).

Top-down propagation

$P(C)$	c_1	c_2
	0.8	0.2



$P(E \mid C)$	c_1	c_2
e_1	0.9	0.7
e_2	0.1	0.3

$P(F \mid E)$	e_1	e_2
f_1	0.9	0.5
f_2	0.1	0.5

$P(D \mid E)$	e_1	e_2
d_1	0.7	0.4
d_2	0.3	0.6

Consider that the only evidence is $F =$ false - initial conditions for the leaf nodes are: $\lambda_F = [1, 0]$ and $\lambda_D = [1, 1]$ (no evidence)

Example - λ propagation

Multiplying the λ vectors by the corresponding CPTs

$$\lambda_F(E) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0.5 \\ 0.1 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.5 \end{bmatrix}$$

$$\lambda_D(E) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Then, $\lambda(E)$ is obtained by combining the messages from its two sons:

$$\lambda(E) = \begin{bmatrix} 0.9 & 0.5 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.5 \end{bmatrix}$$

Propagation to its parent, C:

$$\lambda_E(C) = \begin{bmatrix} 0.9 & 0.5 \end{bmatrix} \begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.86 & 0.78 \end{bmatrix}$$

Example - π propagation

Given that C is not instantiated, $\pi(C) = [0.8 \;\; 0.2]$

Propagate to its son, E, which also corresponds to multiplying the π vector by the corresponding CPT:

$$\pi(E) = \begin{bmatrix} 0.8 & 0.2 \end{bmatrix} \begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.3 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} 0.86 & 0.14 \end{bmatrix}$$

We now propagate to its son D; however, given that E has another son, F, we also need to consider the λ message from this other son, thus:

$$\pi(D) = \begin{bmatrix} 0.86 & 0.14 \end{bmatrix} \times \begin{bmatrix} 0.9 & 0.5 \end{bmatrix} \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} 0.57 & 0.27 \end{bmatrix}$$

Example - Posterior Probabilities

Given the λ and π vectors for each unknown variable, we just multiply them term by term and then normalize to obtain the posterior probabilities:

$$
\begin{aligned}
P(C) &= \begin{bmatrix} 0.8 & 0.2 \end{bmatrix} \times \begin{bmatrix} 0.86 & 0.78 \end{bmatrix} \approx \alpha \begin{bmatrix} 0.69 & 0.16 \end{bmatrix} \\
&\approx \begin{bmatrix} 0.815 & 0.185 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
P(E) &= \begin{bmatrix} 0.86 & 0.14 \end{bmatrix} \times \begin{bmatrix} 0.9 & 0.5 \end{bmatrix} \approx \alpha \begin{bmatrix} 0.77 & 0.07 \end{bmatrix} \\
&\approx \begin{bmatrix} 0.917 & 0.083 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
P(D) &= \begin{bmatrix} 0.57 & 0.27 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \end{bmatrix} \approx \alpha \begin{bmatrix} 0.57 & 0.27 \end{bmatrix} \\
&\approx \begin{bmatrix} 0.675 & 0.325 \end{bmatrix}
\end{aligned}
$$

Analysis

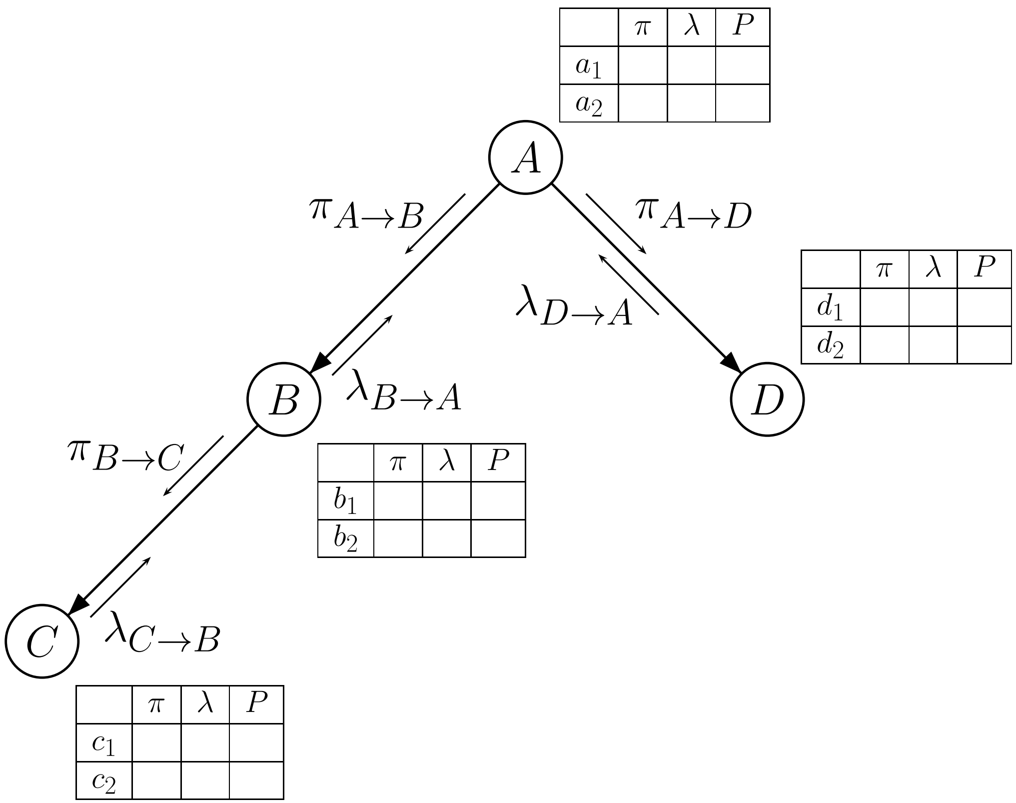
The time complexity to obtain the posterior probability of all the variables in the tree is proportional to the diameter of the network (the number of arcs in the trajectory from the root to the most distant leaf).

The message passing mechanism can be directly extended to polytrees, as these are also singly connected networks. In this case, a node can have multiple parents, so the λ messages should be sent from a node to all its parents

The propagation algorithm only applies to singly connected network

Implementation of Belief Trees

Belief Tree:



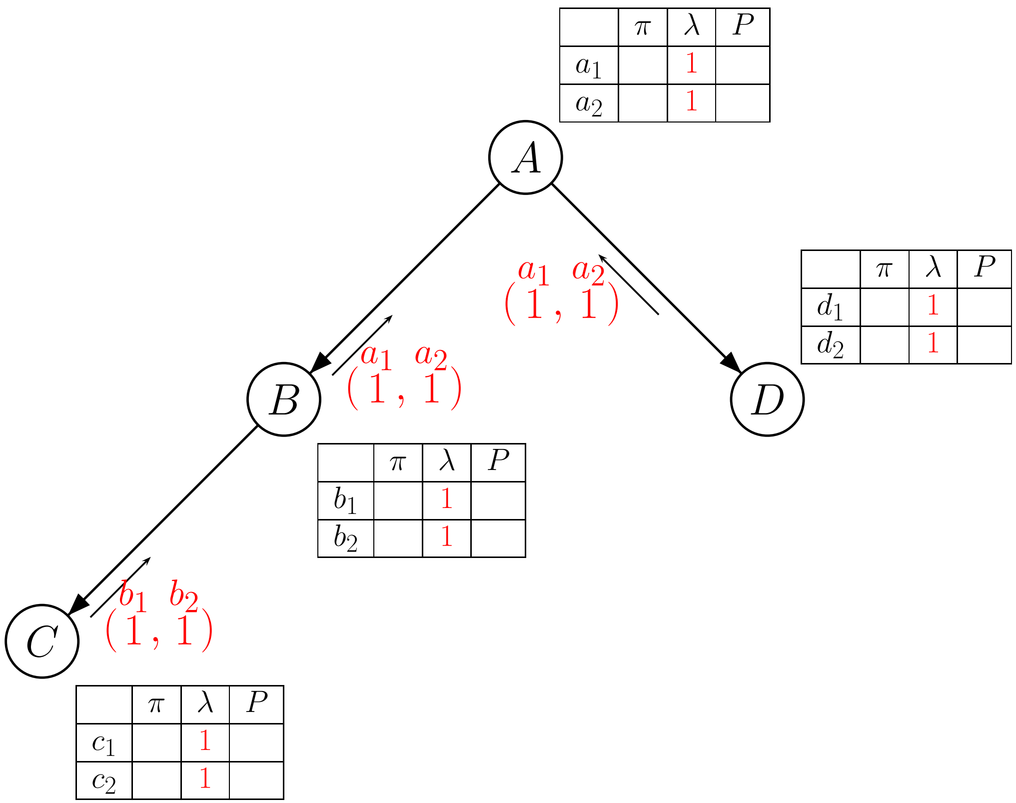
Parameters:

$$P(a_1) = 0.1 \quad P(b_1 \mid a_1) = 0.7$$
$$P(b_1 \mid a_2) = 0.2$$
$$P(d_1 \mid a_1) = 0.8 \quad P(c_1 \mid b_1) = 0.4$$
$$P(d_1 \mid a_2) = 0.4 \quad P(c_1 \mid b_2) = 0.001$$

Desired:

$$P(A) \quad P(B) \quad P(C) \quad P(D)$$

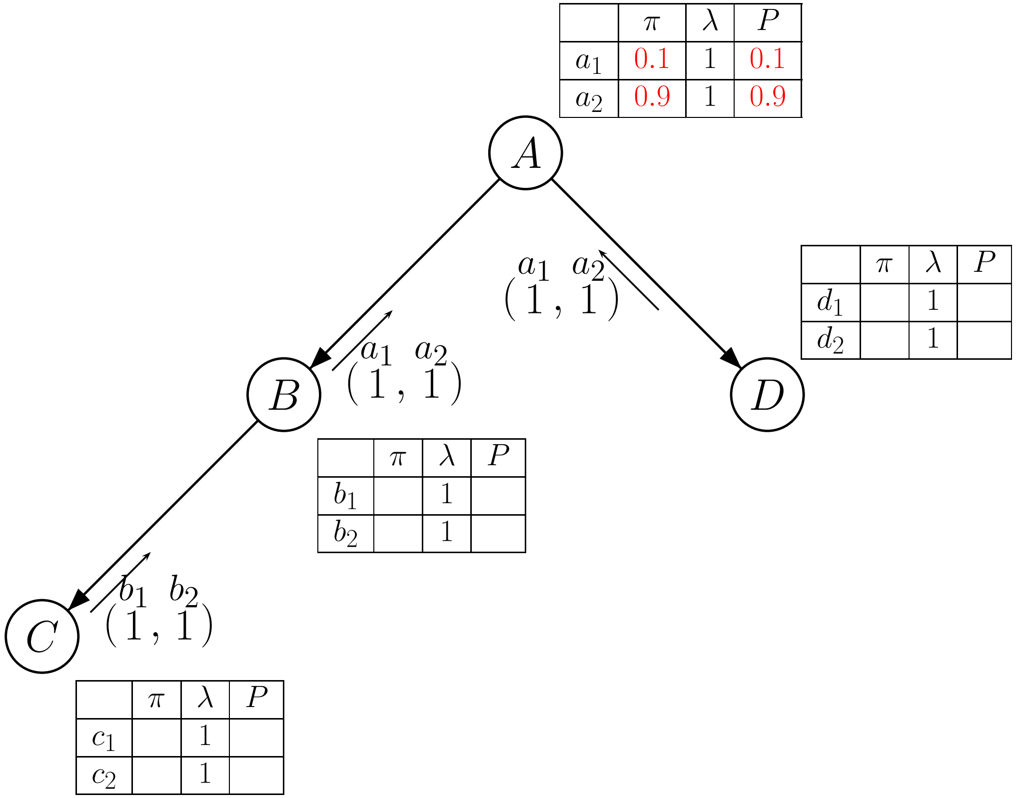
Belief Tree:



Initialization Phase:

Set all λ-messages and λ-values to 1.

Belief Tree:

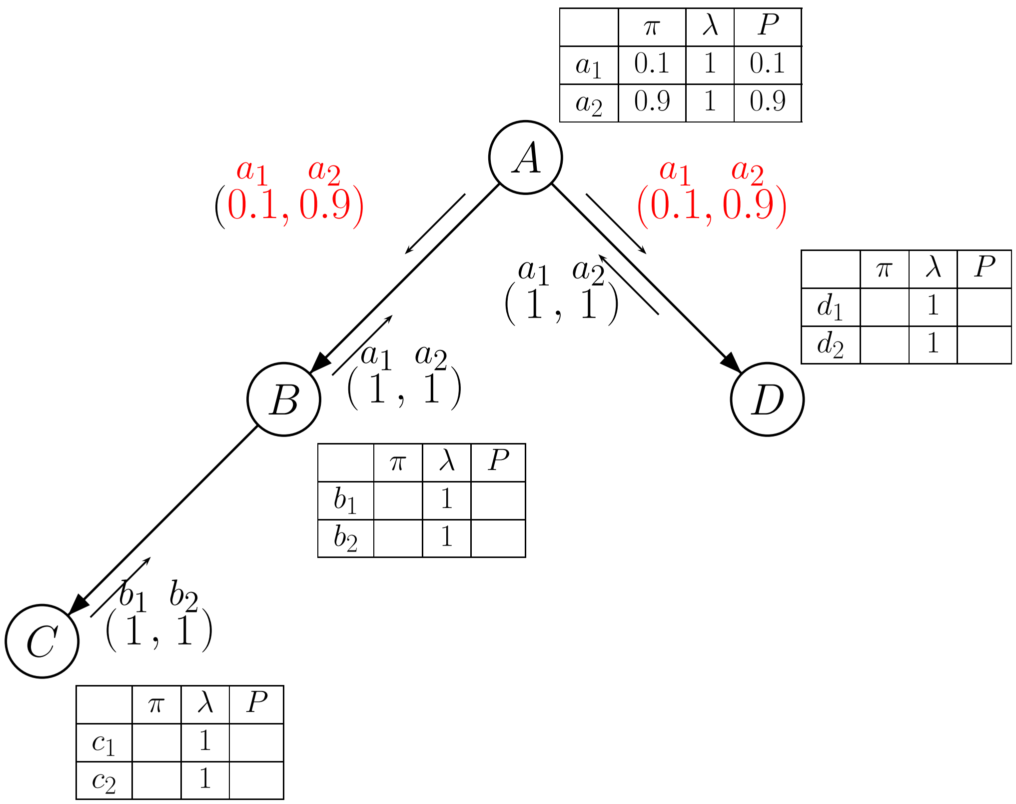


Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$

Implementation of Belief Trees (4)

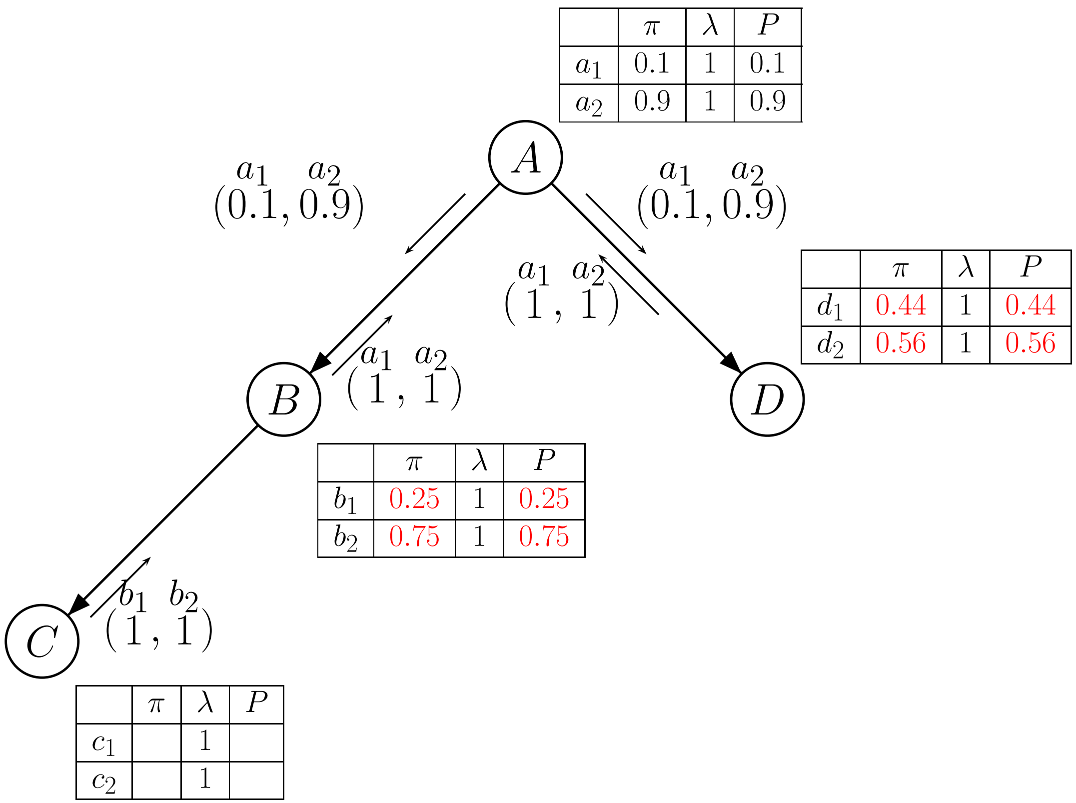
Belief Tree:



Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.

Belief Tree:



	π	λ	P
a_1	0.1	1	0.1
a_2	0.9	1	0.9

	π	λ	P
b_1	0.25	1	0.25
b_2	0.75	1	0.75

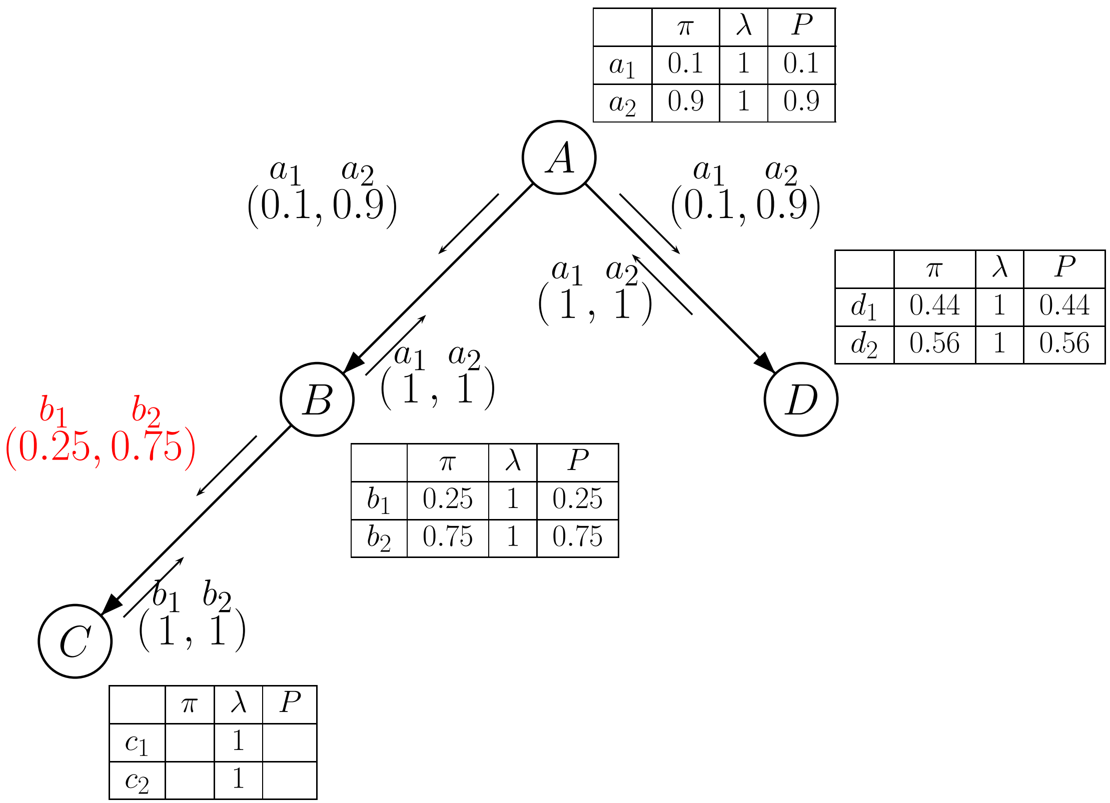
	π	λ	P
c_1		1	
c_2		1	

	π	λ	P
d_1	0.44	1	0.44
d_2	0.56	1	0.56

Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.
B and D update their π-values.

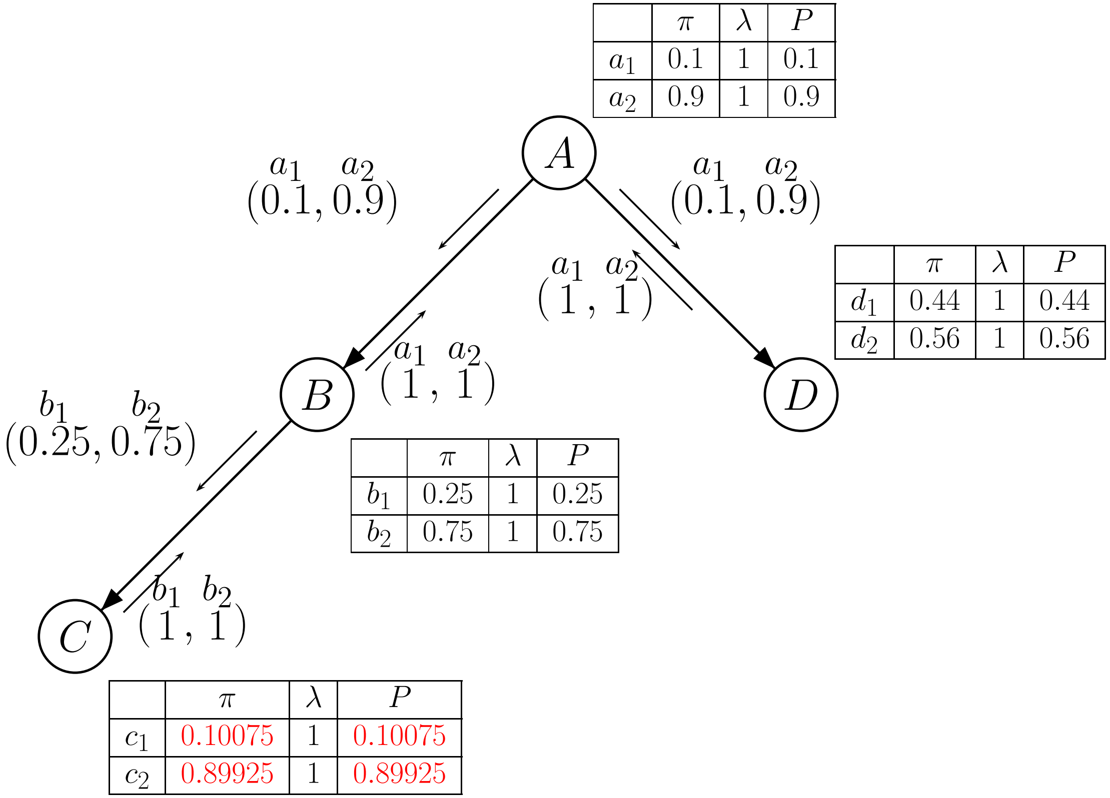
Belief Tree:



Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.
B and D update their π-values.
B sends π-message to C.

Belief Tree:

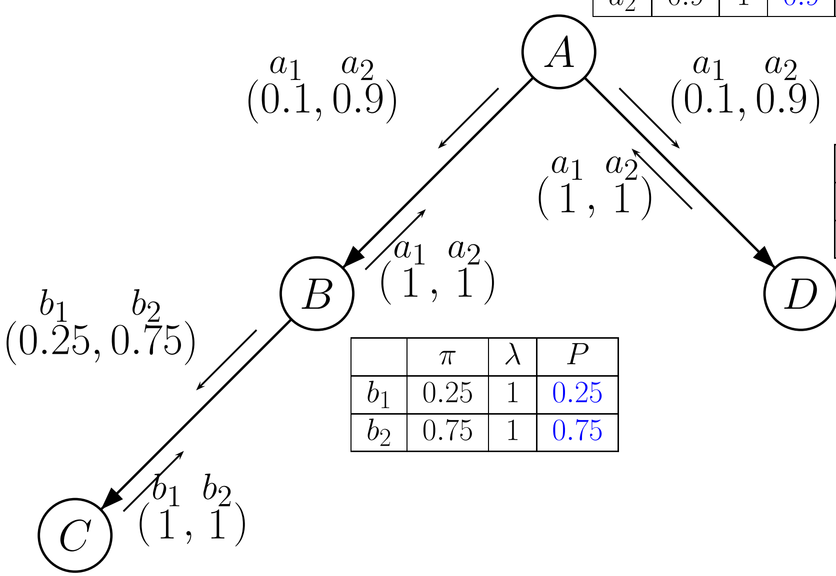


Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.
B and D update their π-values.
B sends π-message to C.
C updates it π-value.

Implementation of Belief Trees (8)

Belief Tree:



	π	λ	P
a_1	0.1	1	0.1
a_2	0.9	1	0.9

	π	λ	P
d_1	0.44	1	0.44
d_2	0.56	1	0.56

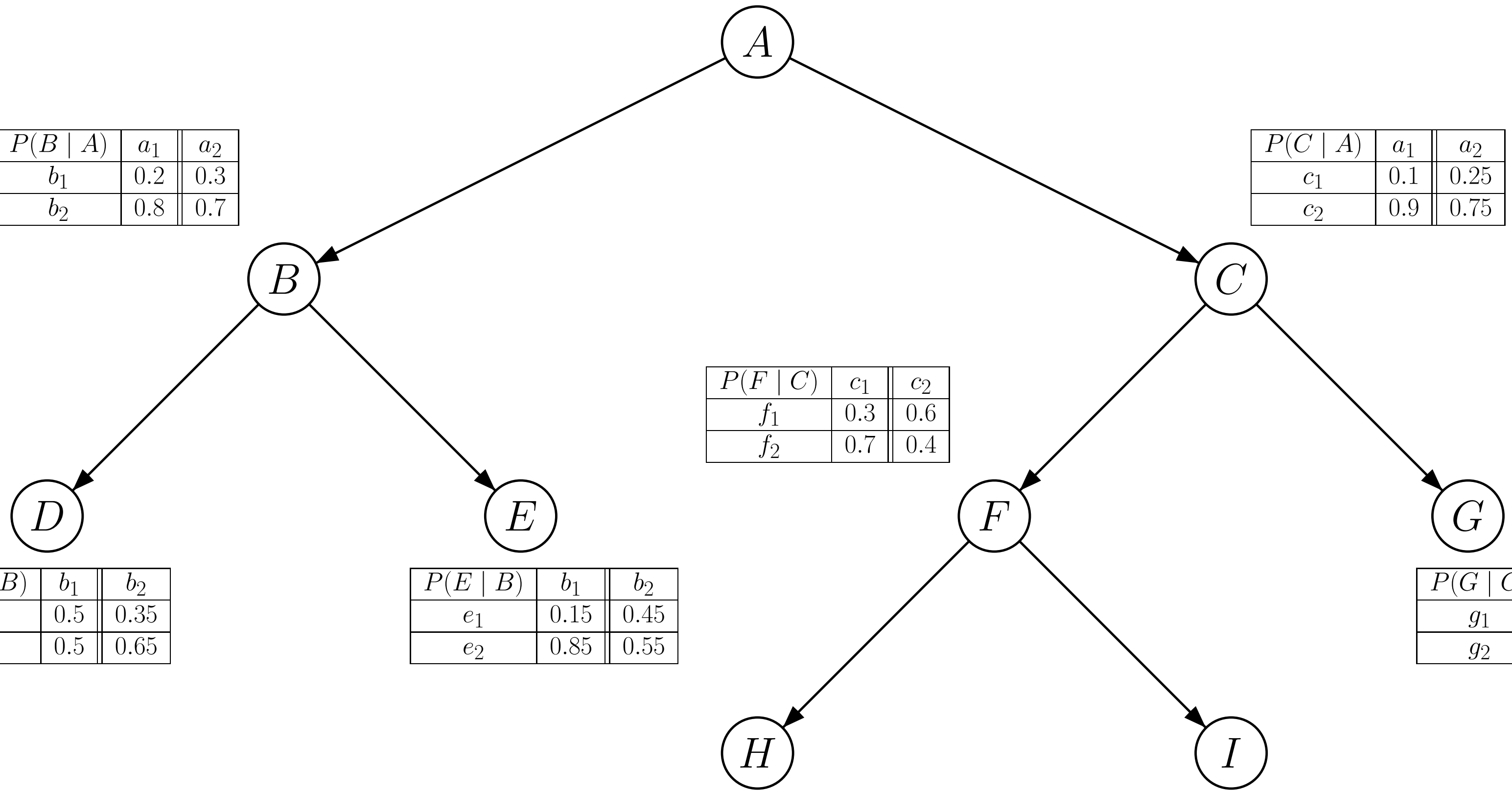
	π	λ	P
b_1	0.25	1	0.25
b_2	0.75	1	0.75

	π	λ	P
c_1	0.10075	1	0.10075
c_2	0.89925	1	0.89925

Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.
B and D update their π-values.
B sends π-message to C.
C updates it π-value.
Initialization finished.

Larger Network (1): Parameters



$P(A)$	$\emptyset$
a_1	0.4
a_2	0.6

$P(B \mid A)$	a_1	a_2
b_1	0.2	0.3
b_2	0.8	0.7

$P(C \mid A)$	a_1	a_2
c_1	0.1	0.25
c_2	0.9	0.75

$P(F \mid C)$	c_1	c_2
f_1	0.3	0.6
f_2	0.7	0.4

$P(D \mid B)$	b_1	b_2
d_1	0.5	0.35
d_2	0.5	0.65

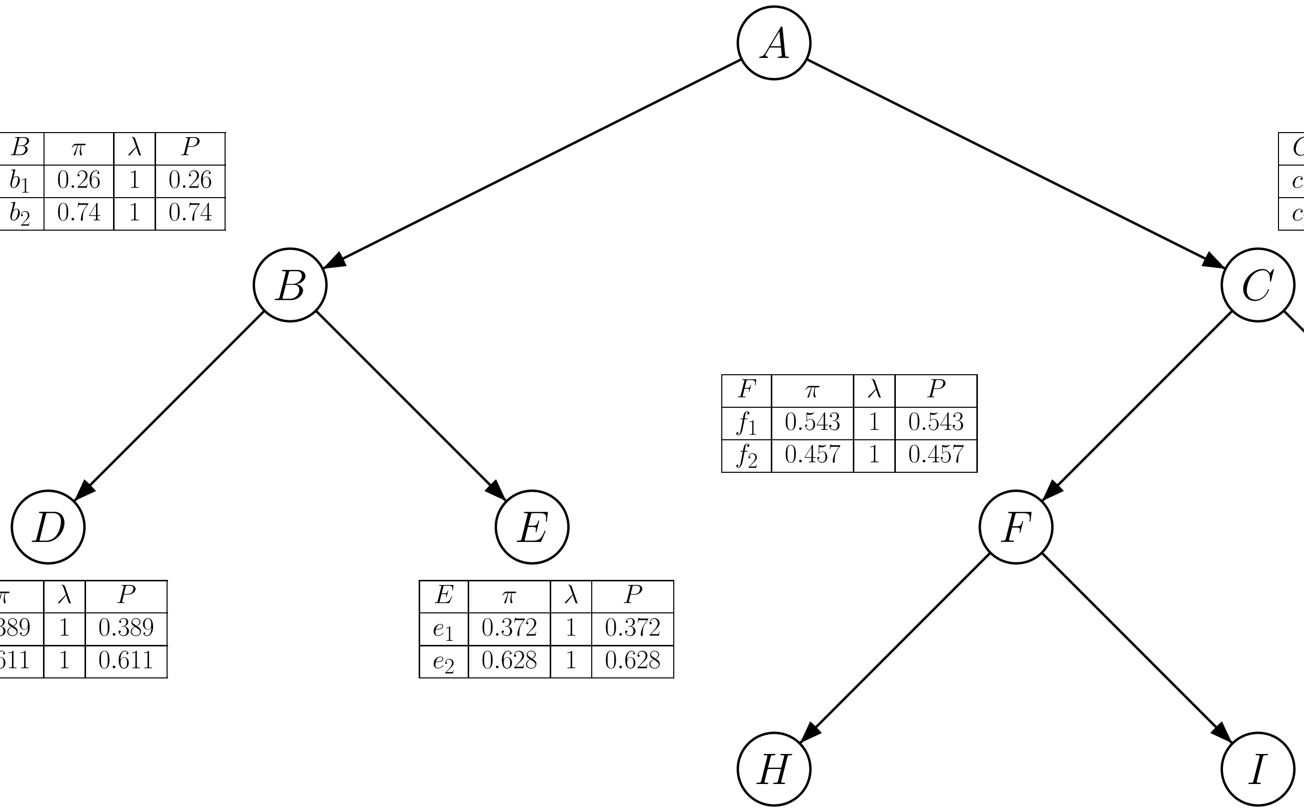
$P(E \mid B)$	b_1	b_2
e_1	0.15	0.45
e_2	0.85	0.55

$P(G \mid C)$	c_1	c_2
g_1	0.25	0.1
g_2	0.75	0.9

$P(H \mid F)$	f_1	f_2
h_1	0.65	0.2
h_2	0.35	0.8

$P(I \mid F)$	f_1	f_2
i_1	0.25	0.5
i_2	0.75	0.5

Larger Network (2): After Initialization



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	1	0.26
b_2	0.74	1	0.74

C	π	λ	P
c_1	0.19	1	0.19
c_2	0.81	1	0.81

F	π	λ	P
f_1	0.543	1	0.543
f_2	0.457	1	0.457

D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

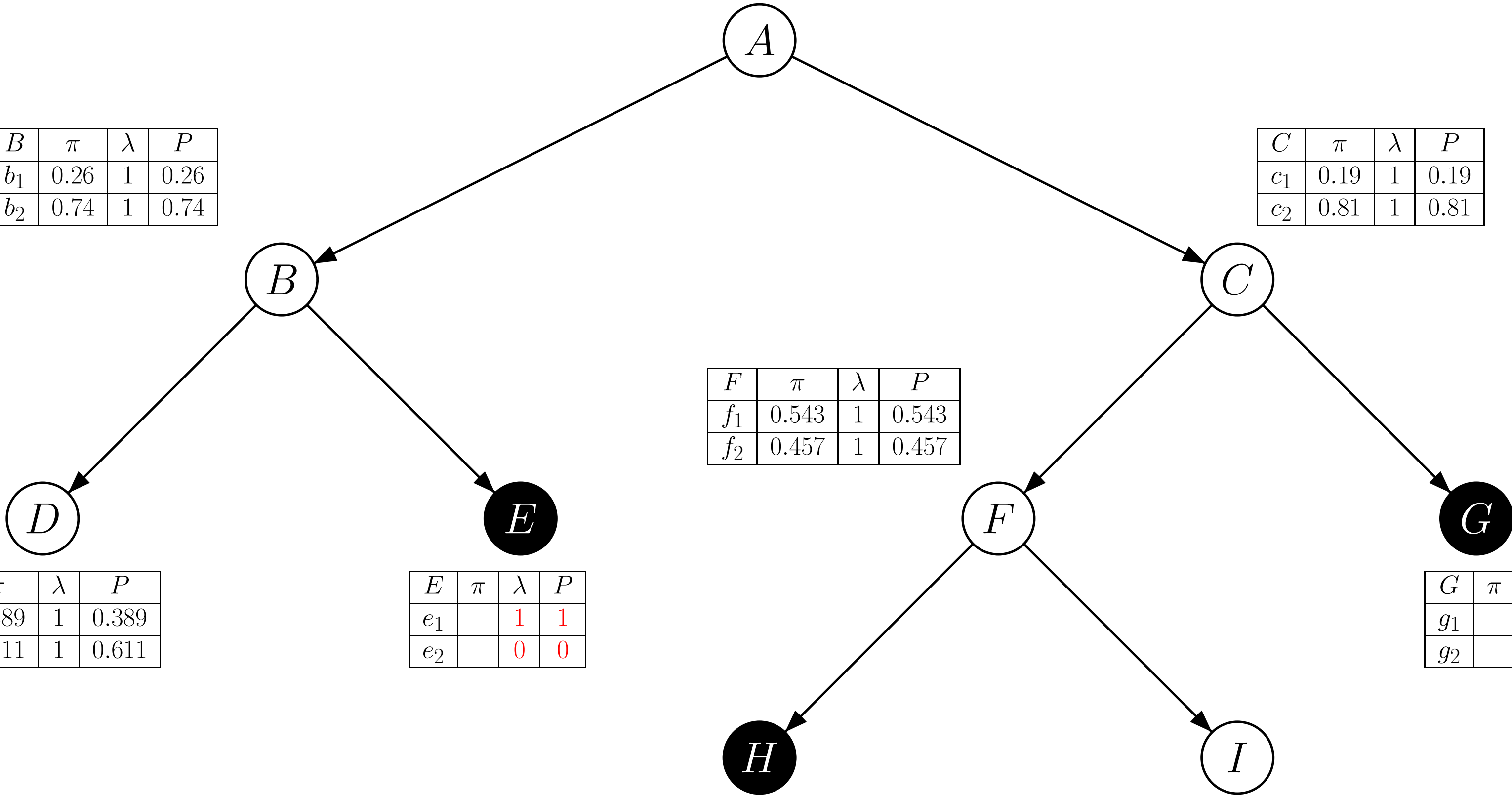
E	π	λ	P
e_1	0.372	1	0.372
e_2	0.628	1	0.628

G	π	λ	P
g_1	0.1285	1	0.1285
g_2	0.8715	1	0.8715

H	π	λ	P
h_1	0.4444	1	0.4444
h_2	0.5556	1	0.5556

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357

Larger Network (3): Set Evidence e_1, g_1, h_1



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	1	0.26
b_2	0.74	1	0.74

C	π	λ	P
c_1	0.19	1	0.19
c_2	0.81	1	0.81

F	π	λ	P
f_1	0.543	1	0.543
f_2	0.457	1	0.457

D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

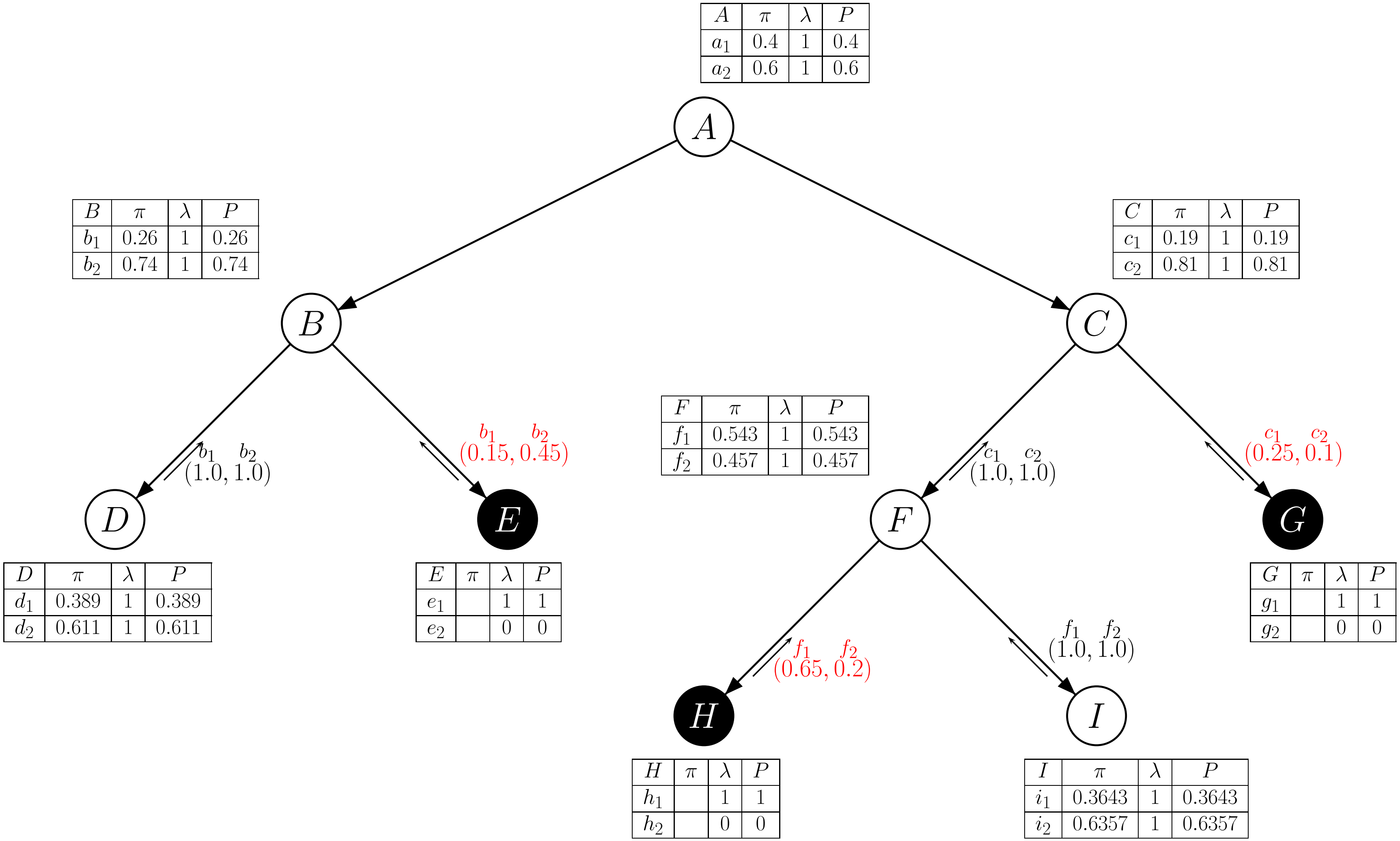
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357

Larger Network (4): Propagate Evidence



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	1	0.26
b_2	0.74	1	0.74

C	π	λ	P
c_1	0.19	1	0.19
c_2	0.81	1	0.81

F	π	λ	P
f_1	0.543	1	0.543
f_2	0.457	1	0.457

D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

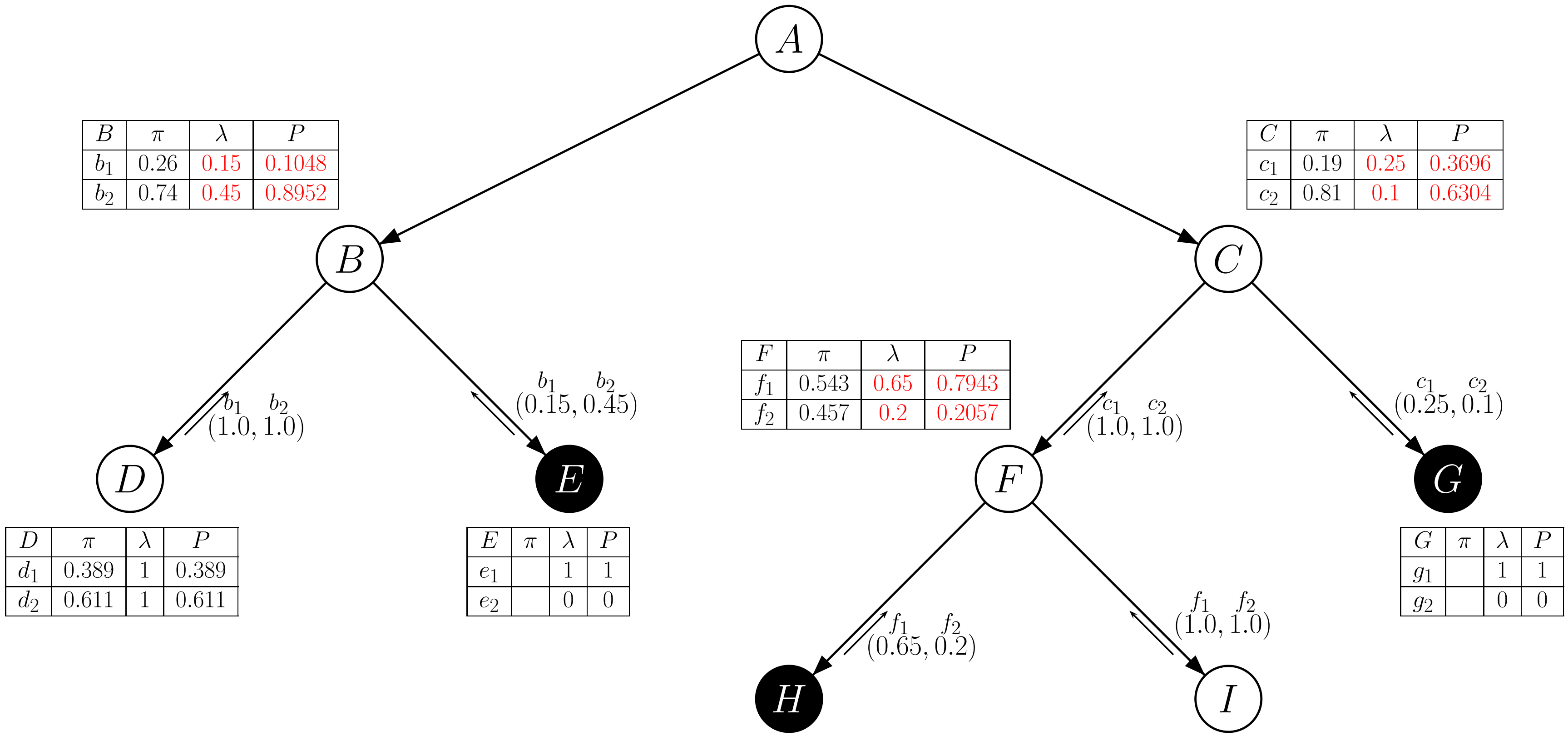
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357

Larger Network (5): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.25	0.3696
c_2	0.81	0.1	0.6304

F	π	λ	P
f_1	0.543	0.65	0.7943
f_2	0.457	0.2	0.2057

D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

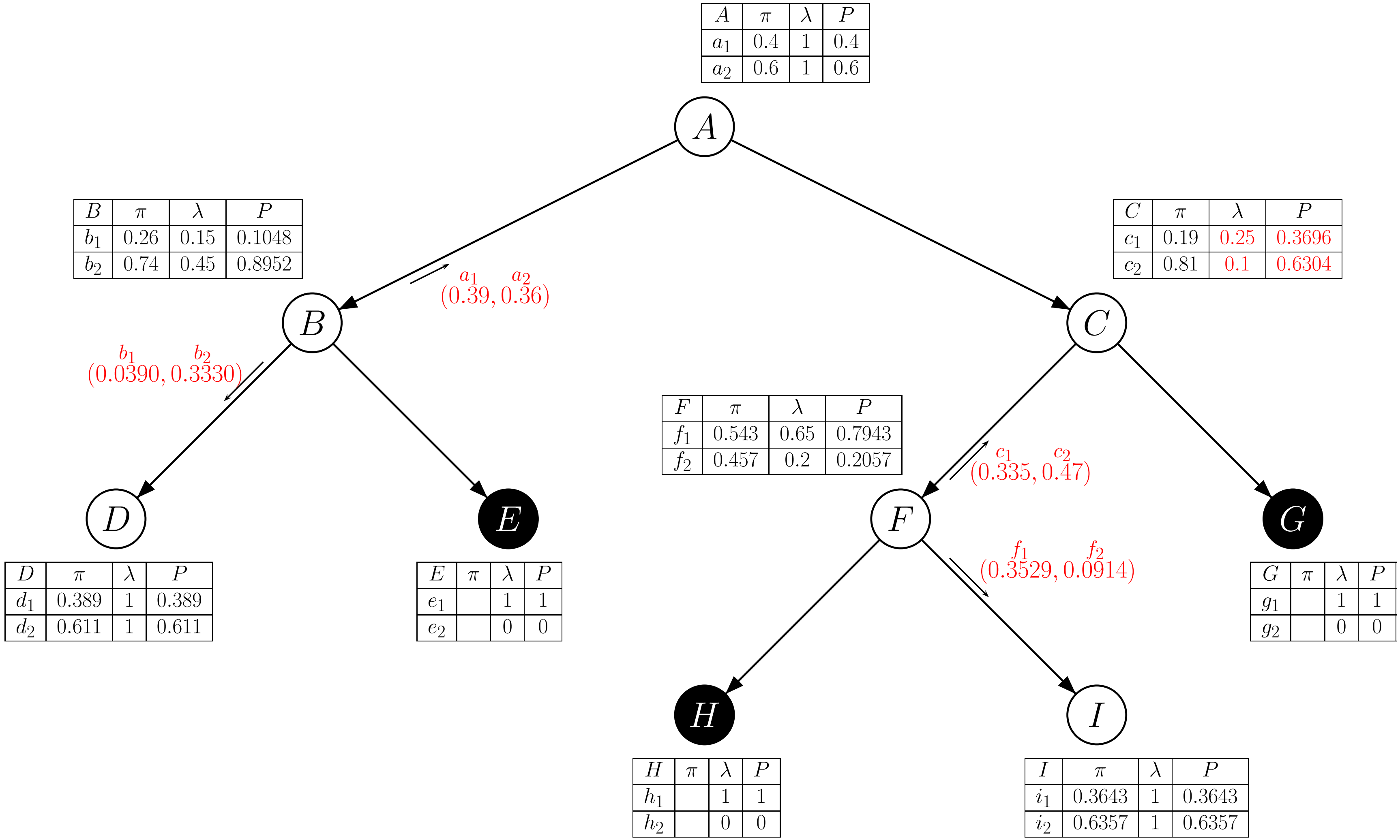
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357

Larger Network (6): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.25	0.3696
c_2	0.81	0.1	0.6304

F	π	λ	P
f_1	0.543	0.65	0.7943
f_2	0.457	0.2	0.2057

D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

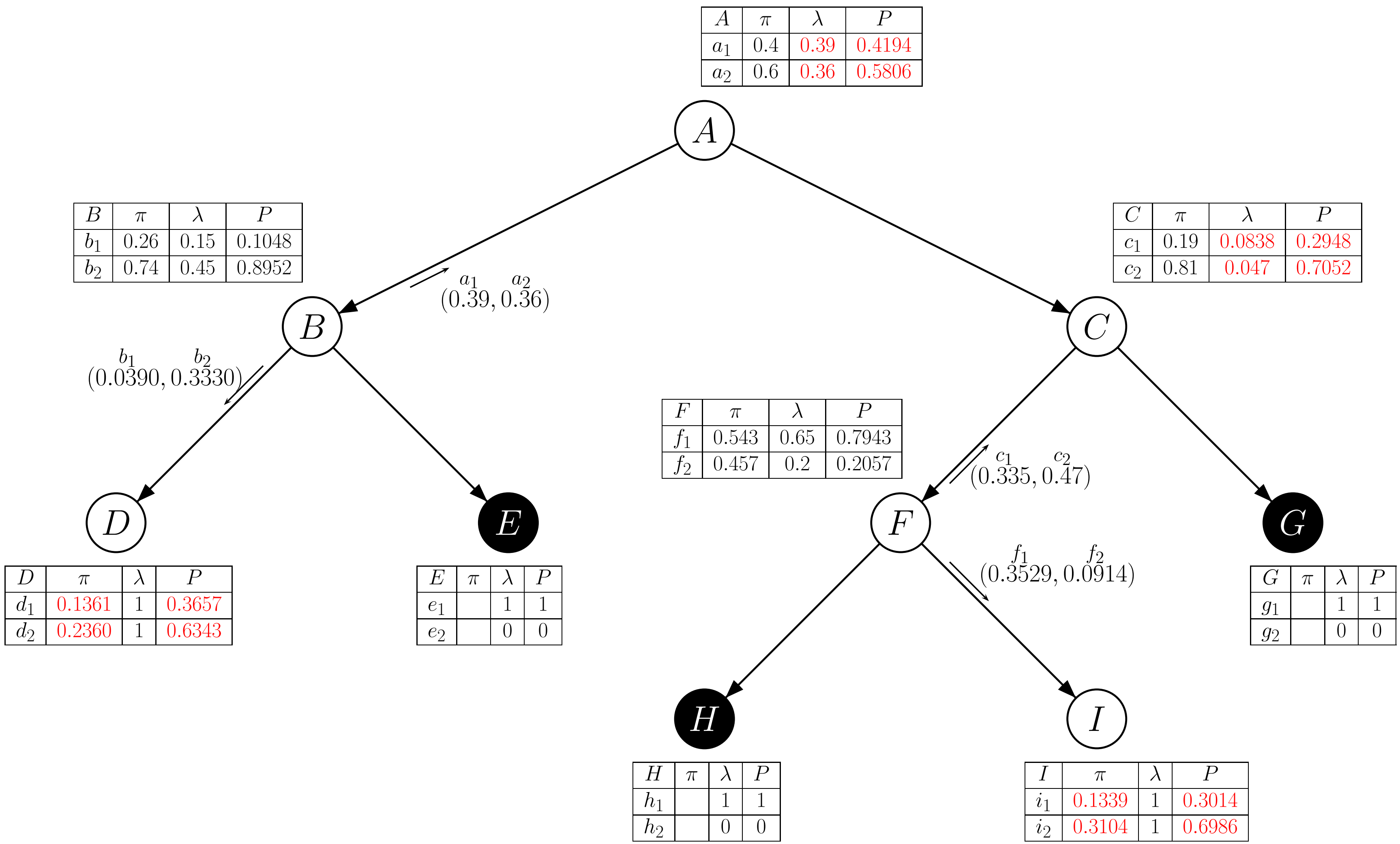
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357

Larger Network (7): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	0.39	0.4194
a_2	0.6	0.36	0.5806

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.0838	0.2948
c_2	0.81	0.047	0.7052

F	π	λ	P
f_1	0.543	0.65	0.7943
f_2	0.457	0.2	0.2057

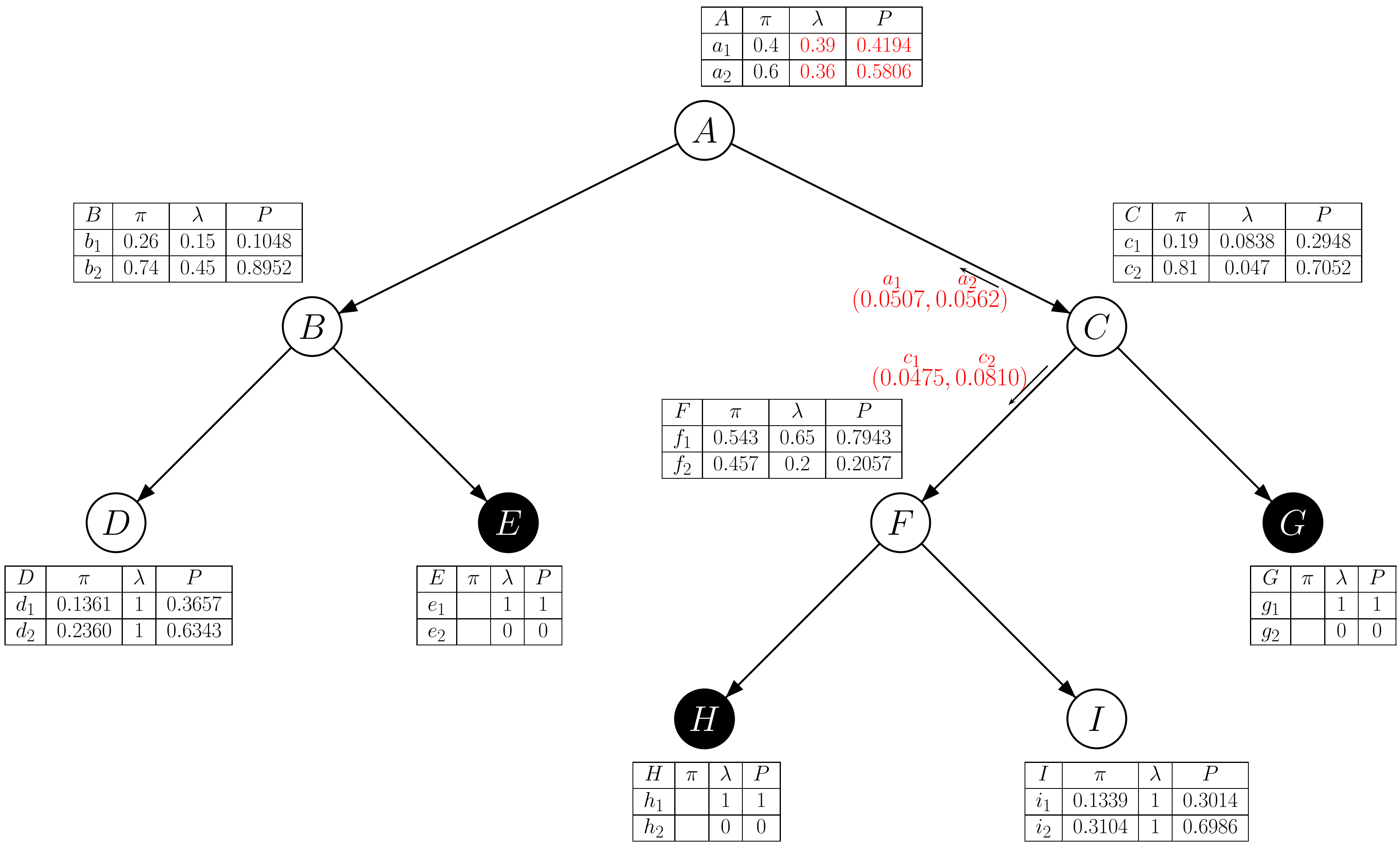
D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.1339	1	0.3014
i_2	0.3104	1	0.6986



A	π	λ	P
a_1	0.4	0.39	0.4194
a_2	0.6	0.36	0.5806

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.0838	0.2948
c_2	0.81	0.047	0.7052

F	π	λ	P
f_1	0.543	0.65	0.7943
f_2	0.457	0.2	0.2057

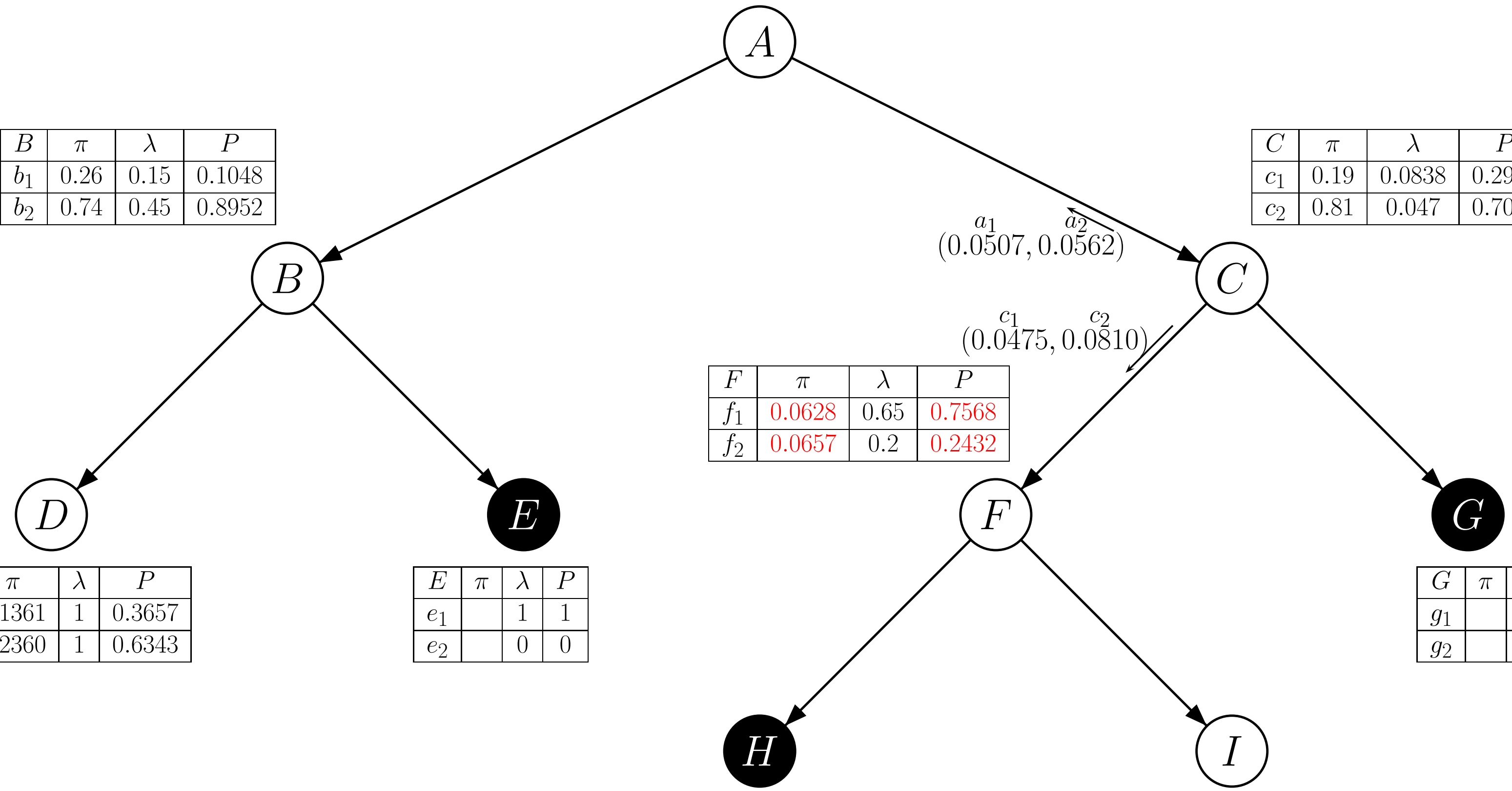
D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.1339	1	0.3014
i_2	0.3104	1	0.6986



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.0838	0.2948
c_2	0.81	0.047	0.7052

F	π	λ	P
f_1	0.0628	0.65	0.7568
f_2	0.0657	0.2	0.2432

D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

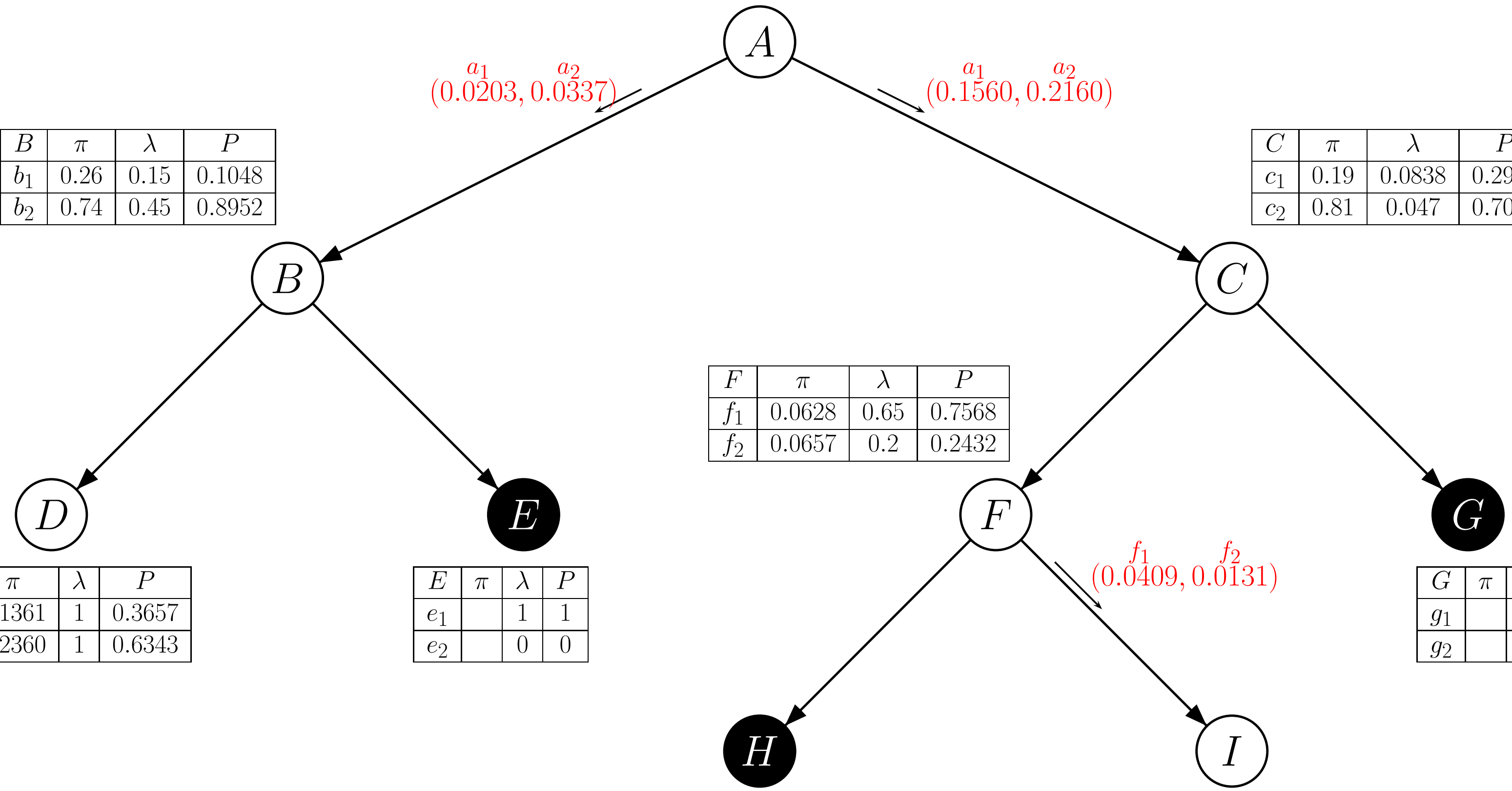
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.1339	1	0.3014
i_2	0.3104	1	0.6986

Larger Network (10): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.0838	0.2948
c_2	0.81	0.047	0.7052

F	π	λ	P
f_1	0.0628	0.65	0.7568
f_2	0.0657	0.2	0.2432

D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

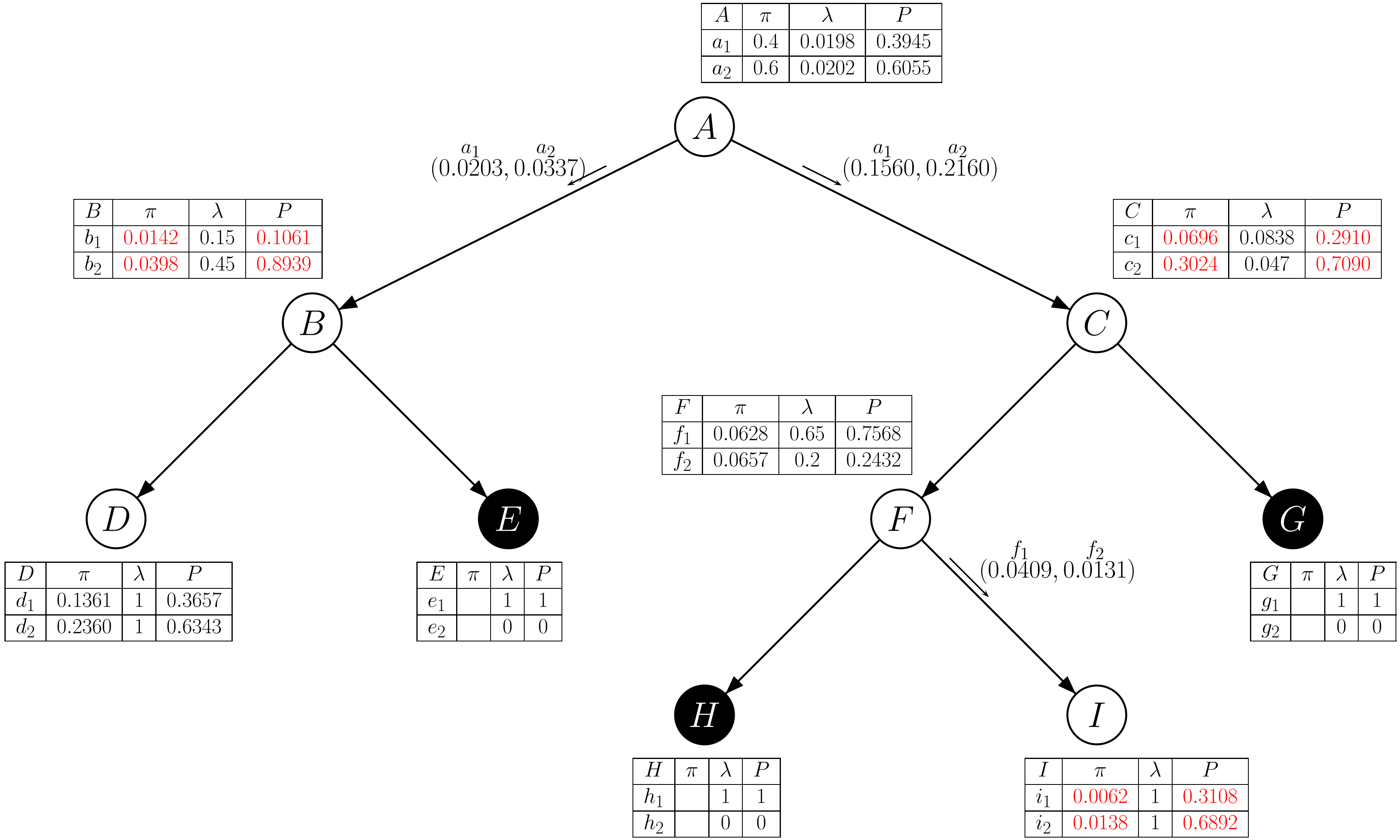
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.1339	1	0.3014
i_2	0.3104	1	0.6986

Larger Network (11): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0628	0.65	0.7568
f_2	0.0657	0.2	0.2432

D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

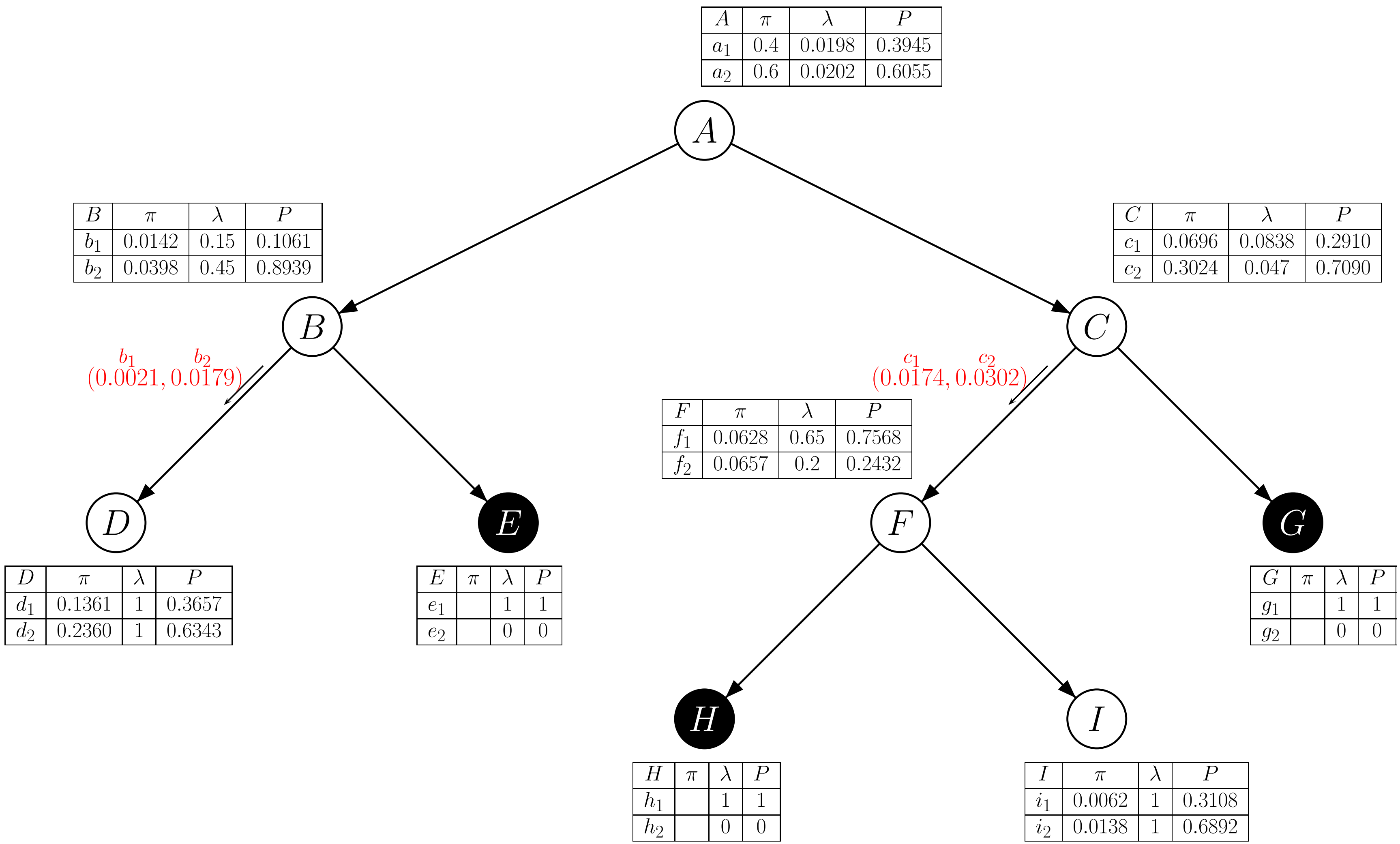
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3108
i_2	0.0138	1	0.6892

Larger Network (12): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0628	0.65	0.7568
f_2	0.0657	0.2	0.2432

D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

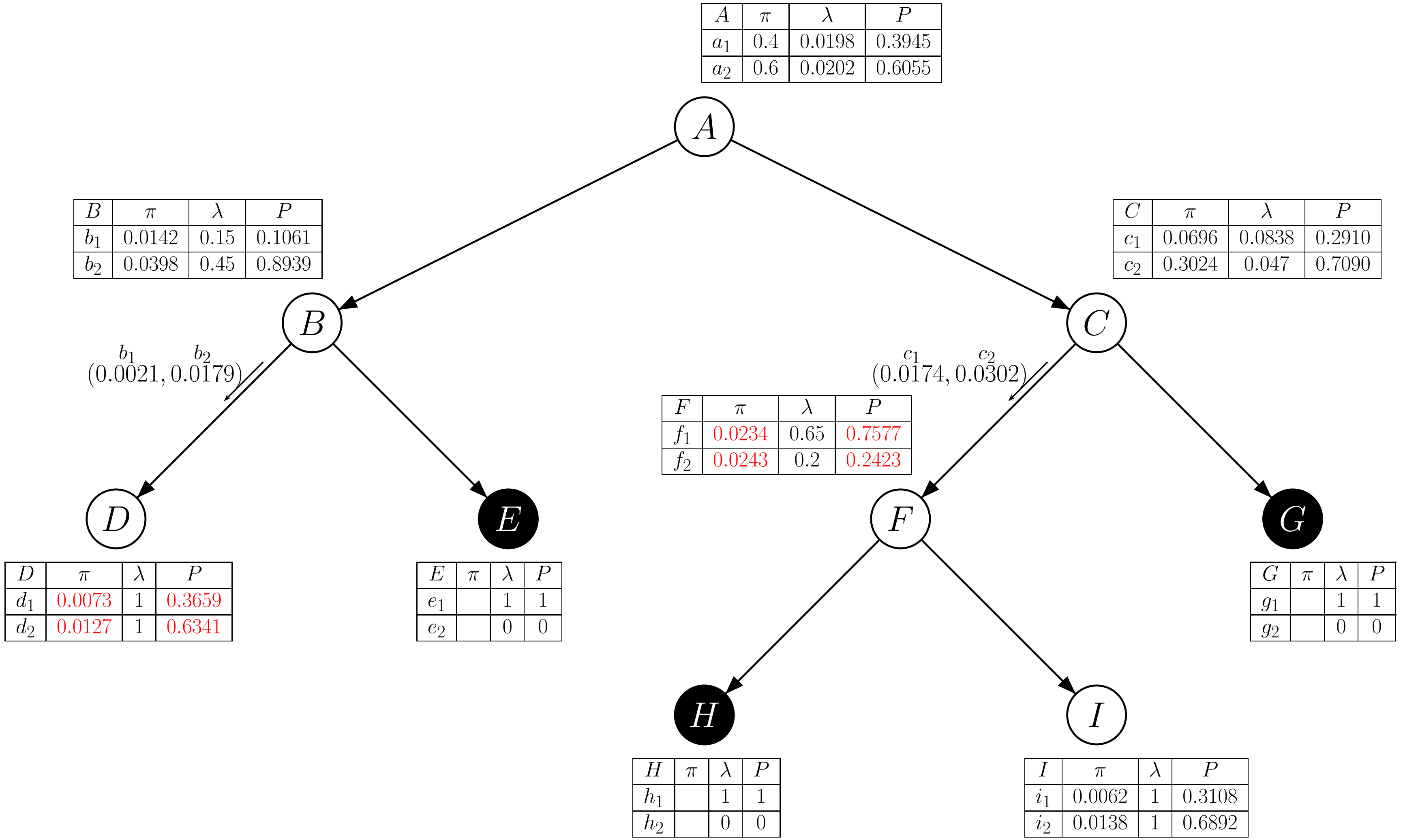
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3108
i_2	0.0138	1	0.6892

Larger Network (13): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0234	0.65	0.7577
f_2	0.0243	0.2	0.2423

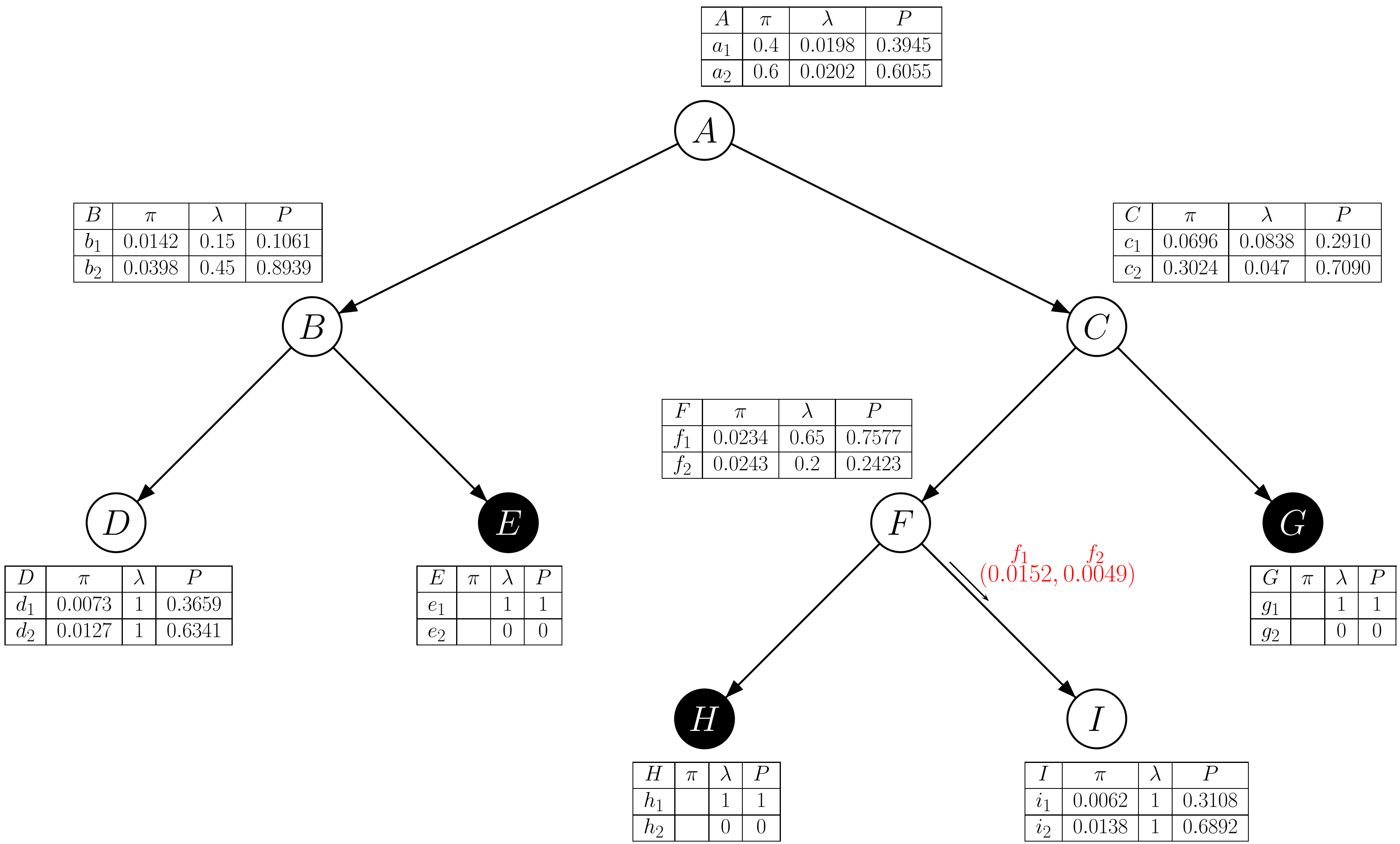
D	π	λ	P
d_1	0.0073	1	0.3659
d_2	0.0127	1	0.6341

E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3108
i_2	0.0138	1	0.6892



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0234	0.65	0.7577
f_2	0.0243	0.2	0.2423

D	π	λ	P
d_1	0.0073	1	0.3659
d_2	0.0127	1	0.6341

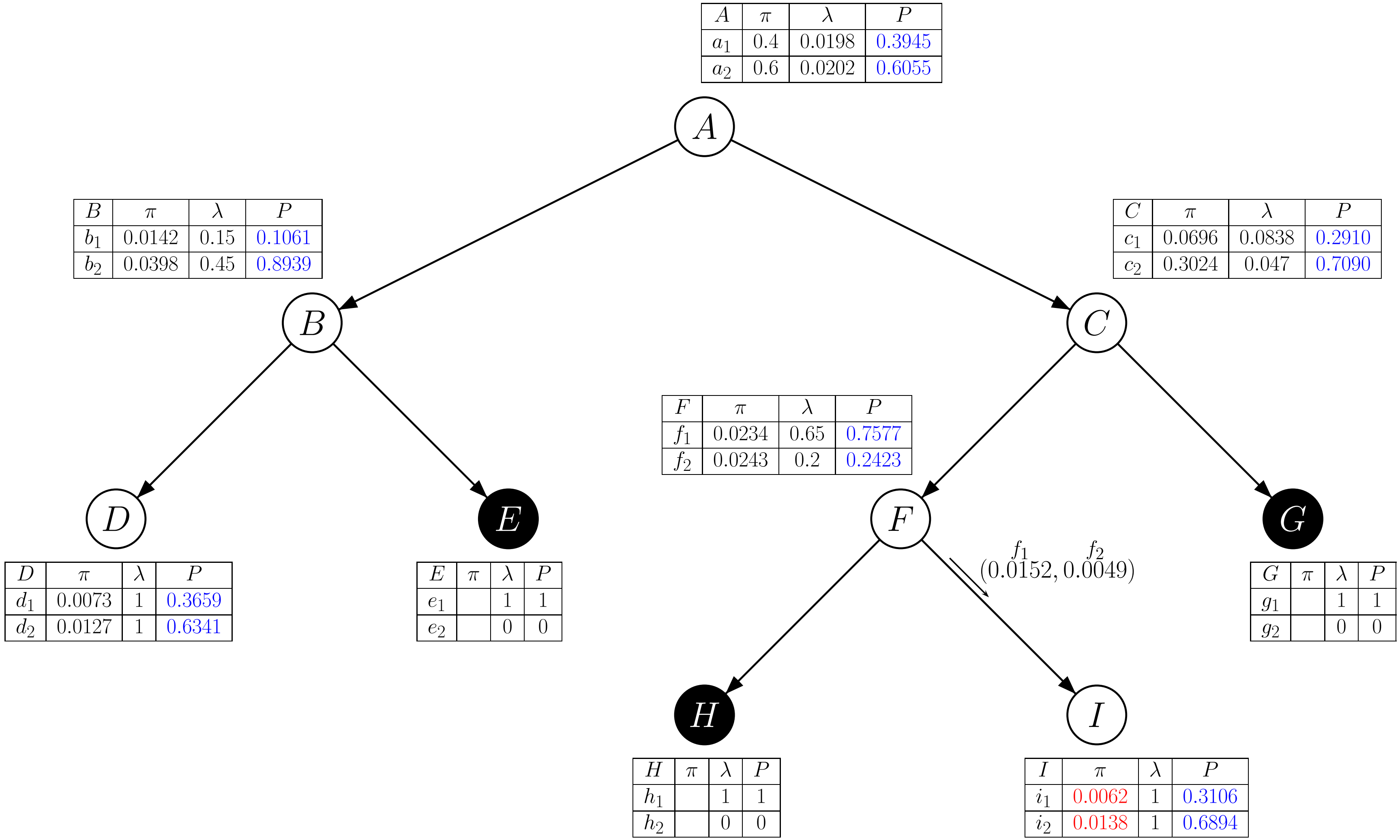
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3108
i_2	0.0138	1	0.6892

Larger Network (15): Finished



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0234	0.65	0.7577
f_2	0.0243	0.2	0.2423

D	π	λ	P
d_1	0.0073	1	0.3659
d_2	0.0127	1	0.6341

E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3106
i_2	0.0138	1	0.6894