Fuzzy Systems
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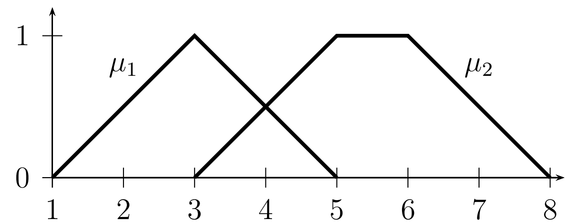
Assignment Sheet 4

Assignment 12 Fuzzy Set Operations

Let the following two fuzzy sets be given:



Compute and draw for each of the pairs

a) the complement of μ_1 *w.r.t.* $U = [1, 8]$ using the standard fuzzy negation,
b) the intersection of μ_1 and μ_2 using the standard fuzzy t-norm $\top_{\text{min}}$,
c) the intersection of μ_1 and μ_2 using the algebraic product $\top_{\text{prod}}$,
d) the intersection of μ_1 and μ_2 using the Łukasiewicz t-norm $\top_{\text{Łuka}}$,
e) the union of μ_1 and μ_2 using the standard fuzzy t-conorm $\perp_{\text{max}}$,
f) the union of μ_1 and μ_2 using the algebraic sum $\perp_{\text{sum}}$,
g) the union of μ_1 and μ_2 using the Łukasiewicz t-conorm $\perp_{\text{Łuka}}$.

Assignment 13 Fuzzy Negation

In order to construct an involutive negation, one can use either a strictly monotonously increasing or decreasing generator function:

Theorem: $\sim: [0, 1] \mapsto [0, 1]$ *is an involutive fuzzy negation if there exists a continuous function* $g : [0, 1] \mapsto \mathbb{R}$ *that fulfills the following properties:*

(i) $g(0) = 0$.

(ii) g *is strictly monotonously increasing.*

(iii) $\sim a = g^{-1}(g(1) - g(a))$.

Theorem: $\sim: [0, 1] \mapsto [0, 1]$ *is an involutive fuzzy negation if there exists a continuous function* $f : [0, 1] \mapsto \mathbb{R}$ *that fulfills the following properties:*

(i) $f(1) = 0$.

(ii) f *is strictly monotonously decreasing.*

(iii) $\sim a = f^{-1}(f(0) - f(a))$.

Now, consider the class of increasing generator functions

$$g_\lambda(a) = \frac{a}{\lambda + (1-\lambda)a}.$$

Apply the given theorem, which allows to construct an involutive fuzzy negation from an arbitrary continuous and strictly increasing function g with $g(0) = 0$. Draw the resulting fuzzy negation for several values of λ.

Assignment 14 Greatest t-norm

Motivate graphically that the Minimum is the greatest t-norm.
Draw a 3D-Plot for two fuzzy truth variables in [0,1] and the corresponding output variable in [0,1] as e.g. done on slide 8 of the lecture on fuzzy logic.

Start drawing the values necessary for fulfilling the crisp logic, then iteratively add the properties of t-norms and their graphical meanings in your drawing.

Assignment 15 Fuzzy Conjunction

Prove the following theorem which was given in the lecture:

Theorem: *For all t-norms $\top$ and all fuzzy truth values $a, b \in [0,1]$ it is*

$$\top_{-1}(a,b) \leq \top(a,b) \leq \top_{\min}(a,b),$$

where $\top_{\min}(a,b) = \min\{a,b\}$ is the standard fuzzy conjunction and $\top_{-1}$ is the so-called drastic product

$$\top_{-1}(a,b) = \begin{cases} a & \textit{if } b = 1, \\ b & \textit{if } a = 1, \\ 0 & \textit{otherwise.} \end{cases}$$