Fuzzy Systems
Prof. Dr. Rudolf Kruse, Alexander Dockhorn

Assignment Sheet 8

Assignment 28 Quantifiers

To describe the concept "x is a small number", let $x \in \mathbb{N} \cup \{0\}$ and two membership functions $\mu_1(x)$ and $\mu_2(x)$ be defined as follows:

$$\mu_1(x) = \begin{cases} \frac{20-x}{20}, & \text{if } x < 20 \\ 0, & \text{otherwise} \end{cases}$$
$$\mu_2(x) = 0.95^x$$

Compute the fuzzy truth value of the proposition "All single-digit numbers of the decimal number system are small" with respect to both μ_1 and μ_2. As conjunction, utilize both

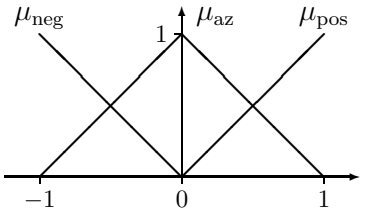
a) $\top_{\min}(a, b) = \min\{a, b\}$,

b) $\top_{\text{prod}}(a, b) = a \cdot b$.

Are the results plausible with regard to your intuition?

Assignment 29 Mamdani-Assilian Controller

Consider a Mamdani-Assilian controller with two inputs $\xi_1 \in X_1 = [-1, 1]$ and $\xi_2 \in X_2 = [-1, 1]$ and one output $\eta \in Y = [-1, 1]$. The utilized fuzzy partitions shall be the same for all three domains. They are shown below on the left ("az" means "approximately zero"). The rule base of the controller is shown on the right in tabular form.



ξ_2 \ ξ_1	neg	az	pos
neg	neg		az
az		az	
pos	az		pos

a) Determine the fuzzy output of this controller for the following input tuples: $(\xi_1, \xi_2) \in \{(0, 0), (0.4, 0.5), (-0.7, 0.9), (-0.5, 0)\}$.

b) Determine crisp output values from the fuzzy outputs computed in part a) using the mean of maxima method and the center of gravity method (for COG an approximation is sufficient, you need not do the exact calculations, which are tedious in some cases.)

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Assignment 30 Mamdani-Assilian Controller

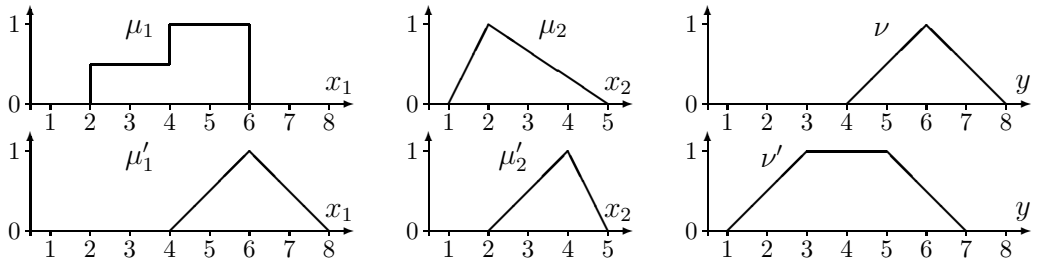
Design a Mamdani-Assilian controller with two inputs $\xi_1 \in X_1 = [0, 3]$ and $\xi_2 \in X_2 = [0, 3]$ and one output $\eta \in Y = [0, 4]$, which uses center of gravity as the defuzzication method. This fuzzy controller should compute the following mappings:

$$(1, 0) \mapsto 2, \quad (1, 3) \mapsto 4,$$
$$(0, 2) \mapsto 2, \quad (2, 2) \mapsto 4,$$
$$(2, 0) \mapsto 2.$$

Try to use as few fuzzy sets as possible. Determine the output of your fuzzy controller for the two input tuples $(1.5, 1.5)$ and $(0.5, 1.5)$.

Assignment 31 Mamdani-Assilian Controller

Let the following fuzzy sets and rules be given:



R_1 : if x_1 is μ_1 and x_2 is μ_2 then y is ν
R_2 : if x_1 is μ_1' and x_2 is μ_2' then y is ν'

a) Based on these fuzzy sets and the rules, which output μ_{output} does a Mamdani-Assilian controller return for the input tuple $(5, 2.5)$?

b) Which crisp output values are (approximately) obtained by defuzzification of the output set with both mean of maxima method and center of gravity method?