Fuzzy Systems
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Assignment Sheet 9

Assignment 32 Takagi-Sugeno Controller

Construct a Takagi-Sugeno controller with two inputs and one output that computes the following (partially defined) function (cf. Assignment 30):

$$\begin{array}{ll} (1,0) \mapsto 2, & (1,3) \mapsto 4, \\ (0,2) \mapsto 2, & (2,2) \mapsto 4, \\ (2,0) \mapsto 2. & \end{array}$$

Determine the output of your controller for the inputs $(1,1)$ and $(1.5,1.5)$.

Assignment 33 Takagi-Sugeno Controller

Consider the following definition of triangular fuzzy numbers

$$\mu_{l,m,r} = \begin{cases} \frac{x-l}{m-l} & \text{if } l \leq x \leq m, \\ \frac{r-x}{r-m} & \text{if } m \leq x \leq r, \\ 0 & \text{otherwise} \end{cases}$$

whereas $l, m, r \in \mathbb{R}$ and $l < m < r$. Now, let a Takagi-Sugeno controller with the rule base be given as follows

R_1 **: if** x **is** μ_1 **then** $y = 2$**,**
R_2 **: if** x **is** μ_2 **then** $y = x$**,**
R_3 **: if** x **is** μ_3 **then** $y = 3 - x^2$**,**

whereas $x \in X = [0, 8]$ and X is partitioned by $\mu_1 = \mu_{0,2,4},\ \mu_2 = \mu_{2,4,6},\ \mu_3 = \mu_{4,6,8}$.

a) Compute the output of the controller by using the weighted sum

$$f(x) = \frac{\sum_{r=1}^{3} \mu_{R_r}(x) \cdot f_{R_r}(x)}{\sum_{r=1}^{3} \mu_{R_r}(x)},$$

whereas $\mu_{R_r}(x)$ is the degree of fulfillment that the rule R_r "fires", and f_{R_r} is the output of the rule R_r.

b) Draw the output into a diagram.

Assignment 34 Fuzzy Clustering

Consider the one-dimensional data set

$$1, 3, 4, 5, 8, 10, 11, 12.$$

We want to process this data set with fuzzy c-means clustering using $c = 2$ (two clusters) and the fuzzifier $m = 2$. Assume that the cluster centers are initialized to 1 and 5. Execute one step of alternating optimization as it is used for fuzzy clustering, *i.e.*

a) Compute the membership degrees of the data points for the initial cluster centers.

b) Compute new cluster centers from the membership degrees that have been obtained before.

Assignment 35 Fuzzifier m

Consider the objective function of fuzzy clustering with a fuzzifier $m \geq 1$, *i.e.*

$$J_f(X, U, C) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m d^2(\mathbf{c}_i, \mathbf{x}_j) \quad \text{subject to} \quad \forall j \in \{1, \ldots, n\} : \sum_{i=1}^{c} u_{ij} = 1.$$

Assume that the minimum of J_f is obtained $\forall i \in \{1, \ldots, c\} : \forall j \in \{1, \ldots, n\} : d(\mathbf{c}_i, \mathbf{x}_j) > 0$, *i.e.* the cluster centers do not coincide with any data points.

a) Show that if the fuzzifier $m = 1$ one obtains hard/crisp assignments of data points even if the membership degrees $u_{ij} \in [0, 1]$. Thus, show that the minimum of J_f is attained $\forall i \in \{1, \ldots, c\} : \forall j \in \{1, \ldots, n\} : u_{ij} \in \{0, 1\}$.

b) Show that if the fuzzifier $m > 1$ one cannot obtain hard/crisp assignments of data points even if the membership degrees $u_{ij} \in [0, 1]$. Thus, show that the minimum of J_f is attained $\forall i \in \{1, \ldots, c\} : \forall j \in \{1, \ldots, n\} : u_{ij} \in \,]0, 1[$.

Hint: You may find it easier to consider the special case $c = 2$ (two clusters) and to examine the term for a single data point $\mathbf{x}_j$.