Modified Crowding Distance and Mutation for Multimodal Multi-Objective Optimization

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ABSTRACT

Many real-world applications contain complex optimization problems with several conflicting objectives. Finding a solution which can satisfy all the objectives is usually a challenging task for optimization algorithms. When dealing with these complex multi-objective problems, decision-makers want to find the best trade-off between the conflicting objectives. Another challenge occurs in problems where multiple configurations of the input variables might yield the same objective function values. Such problems are called multimodal problems. For a decision maker, it might be of importance to obtain enough information about all the alternative optimal solutions that reach the same objective value. Traditionally, Evolutionary Algorithms make use selection processes based only on objective function values, which might be a disadvantage when faced with multimodal problems. In this article, we present two operators to use in multimodal multi-objective algorithms, namely a modified crowding distance operator and a neighbourhood Polynomial mutation, which take into account the distribution of solution in the decision space at run-time. Our experimental results demonstrate that the proposed operators are able to outperform the performance of a state-of-the-art method on six current multimodal benchmark functions.

CCS CONCEPTS

• Theory of computation → Evolutionary algorithms; **• Computing methodologies → Genetic algorithms**;

KEYWORDS

Multi-modality, Multi-modal problems, Multi-objective Optimization, Evolutionary Algorithms, Non-dominated Sorting Genetic Algorithm

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1 INTRODUCTION

In the field of multi-objective optimization algorithms, there is a concept of multi-modality that is related to existence of distinct set of solutions in decision space known as Pareto-Set (PS) that is mapped to the same set of solutions in objective space known as Pareto-Front (PF). The main focus of this work is to find and maintain all the sub-set of solutions for PS in multi-modal multi-objective problems. In recent years, there has been growing interests on finding multi-modal solutions for multi-objective optimization problems [5, 8]. In order to find all the sub-set of solutions in decision space, the diversity of the solutions in decision space needs to increase, through niching techniques such as the Crowding distance method. This well-known niching approach is already used in the literature to improve the diversity of solutions in objective space [2]. Although this procedure may lead to a better approximation of the PF, preservation of all the solution in the PS is not guaranteed. Therefore, we adopt the concept of crowding distance in both spaces to obtain a better approximation of the PS and PF. This approach is called *WSCD* as it is computed as the weighted sum of the crowding distance in objective and decision space.
It is a modified version of the crowding distance approach which was applied in the Omni-optimizer algorithm [3], as well as Mo-Ring-PSO-SCD algorithm [8]. The proposed change lies in the assignment of a final crowding distance value for each of the solutions. In the proposed method the obtained final crowding distance value is based on the weighted sum, where different weights are associated with the crowding distance in objective and decision space. The presented method together with an adopted version of the polynomial mutation operator based on the concept provided by Qu et al. in 2012 [1] is applied on the NSGA-II algorithm as an example.

2 PROPOSED METHOD

In this section, we briefly explain the *NSGA-II with Weighted Sum Crowding Distance and Neighborhood Mutation* (NSGA-II-WSCD-NBM) algorithm. The major modification of this algorithm is provided by (1) applying crowding distance as a selection scheme in both decision and objective space and (2) applying a neighborhood structure on polynomial mutation. For the WSCD method, the crowding distance values for each solution are computed both in (first) objective and (then) decision spaces. The final crowding distance value for each the solutions are obtained as a weighted sum, using pre-defined weights assigned to crowding distance values in both spaces. To perform the second modification we applied neighborhood concept on polynomial mutation that is considering

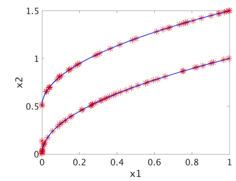
as one of the most important operators [4]. In this method, the adjacent solutions are categorized into groups, and the mutation operator is applied on each of these groups.
The main roll of the adopted mutation operator is to increase diversity of solutions and help get rid of the local optima trap by providing the possibility of doing more mutations for the solutions located in denser areas. Furthermore, the modified crowding distance method is applied to maintain the different sets of solutions that are already found.

3 EXPERIMENTS

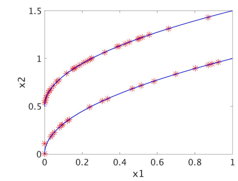
Given the limitation on article length, the experiment settings will be listed without any further details. The reported result are obtained through 31 independent runs where the population is set to 100. Furthermore, in the present work, weights attributed to the crowding distances in the decision and objective space are set to the same value, i.e. $C_1 = C_2 = 0.5$. We use the NSGA-II and Mo-Ring-PSO-SCD as a state-of-the-art algorithm and compare them with the proposed algorithm. We apply the algorithms on 6 test different test problems with to evaluate the performance of the presented algorithms [6, 8]. The codes for NSGA-II is provided by Matlab-based platform, "PlatEMO" [7]. Moreover, we make the experimental results for Mo-Ring-PSO-SCD according to the source code provided by the corresponding author [8]. The experimental results (median and IQR) for the comparison of the used algorithms concerning PSP performance indicator [8] is shown in Table 1. This indicator represents the overlap ratio between the obtained solution set and PS. This indicator is calculated by the division of the Cover Rate (CV) and the IGDX value [9] $PSP = CR/IGDX$. In this formula the CR value represents the maximum spread of the obtained solutions in decision space. A higher CR value shows a better overlap ratio between the bounding box of the obtained set and the PS.
The Mann-Whitney U statistical test is applied on the obtained results, to showcase the superiority of the proposed algorithm. A difference between two results is regarded as significant for values of $p < 0.01$. The best values are highlighted in a bold font according to the outcome of the mentioned test, and significance is shown by an asterisk in the respective columns. The presented results clearly point out the enhanced performances of the proposed algorithm compared to the other approaches, in terms of both the diversity and convergence of obtained solutions. In order to provide a better insight the obtained Pareto set and fronts for the problem MMF3 are visualized in Figure 1.

Table 1: PSP values of different algorithms. An asterisk indicates statistical significance compared to the respective best algorithm

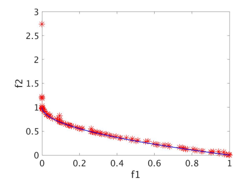
	NSGA-II-WSCD-NBM	Mo-Ring-PSO-SCD	NSGA-II
SSUF1	**15.70245 (4.93927E-1)**	13.143474 (1.637382) *	9.03637 (1.322887) *
SSUF3	**59.214581 (9.175856)**	24.840769 (10.041116) *	10.996908 (6.171965) *
MMF3	**67.170309 (5.562562)**	30.75947 (10.712036) *	12.313134 (6.294302) *
MMF4	**23.973064 (2.83035)**	21.707866 (2.824505) *	8.726443 (2.44963 *
MMF5	**8.722547 (3.52325E-1)**	7.885011 (5.74091E-1) *	5.250953 (1.623423) *
MMF6	**10.037834 (6.13869E-1)**	9.226296 (8.09602E-1) *	4.544513 (1.981098) *



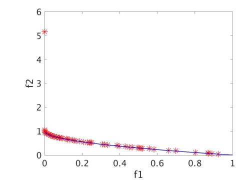
(a) PS for NSGA-II-WSCD-NBM



(b) PS for Mo-Ring-PSO-SCD



(c) PF for NSGA-II-WSCD-NBM



(d) PF for Mo-Ring-PSO-SCD

Figure 1: Obtained solutions in decision and objective space for MMF3 problem

4 CONCLUSION

To solve multimodal multi-objective optimization problems, we propose a modified crowding distance and mutation operators and take NSGA-II algorithm as an example and apply the two proposed modifications on it. The results demonstrate the superiority of the proposed algorithm in compared with the state-of-the-art algorithm. In our future work, we will aim to investigate the effect of different weights for the WSCD method to find the best combination of these constants.

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