Combining Manhattan and Crowding distances in Decision Space for Multimodal Multi-objective Optimization problems

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Summary
This paper presents a new variant of the Non-dominated Sorting Genetic Algorithm to solve Multimodal Multi-objective optimization problems. We introduce a novel method to augment the diversity of solutions in decision space by combining the Manhattan and Crowding distances. In our experiments, we use six test problems with different levels of complexity to examine the performance of our proposed algorithm. The results are compared with NSGA-II and NSGA-II-WSCD algorithms. Using IGDX and IGD performance indicators, we demonstrate the superiority of our proposed method over the rest of competitors to provide a better approximation of the Pareto Set while not getting much worse results in objective space.

Keywords: *Manhattan distance, Crowding distance, Multi modality, Evolutionary Algorithms, Non-dominated Sorting Genetic Algorithm, Multi-objective Optimization.*

1 Introduction

In real world, there are many Multi-objective Optimization Problems (MOP) with at least two conflicting objectives in nature. This means that improving one of the objectives leads to deteriorating the value for the other objectives. Without loss of generality, a multi-objective minimization problems is formulated as follows:

$$\begin{aligned} \text{minimize} \quad & \vec{f}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), ..., f_M(\vec{x})) \qquad (1) \\ \text{subject to} \quad & \vec{x} \in S \subset \mathbb{R}^D \\ & g_i(\vec{x}) \leq 0, i = 1,2,...,G \\ & h_j(\vec{x}) = 0, j = 1,2,...,H \end{aligned}$$

where $\vec{x} = (x_1, x_2, ..., x_D)$ is considered as a D–dimensional decision vector and $(f_1, f_2, ..., f_M)$ is a M–dimensional objective vector. $g_i(\vec{x})$ and $h_j(\vec{x})$ are inequality and equality constraint functions in decision space.

In order to deal with these problems, the concept of domination can be used. Given two vectors $\vec{x}$, $\vec{y} \in S$, $\vec{x}$ is said to be dominated by $\vec{y}$ (denoted by $\vec{y} \prec \vec{x}$) if and only if $\forall j \in \{1,..,M\}$, $f_j(\vec{y}) \leq f_j(\vec{x})$, and $\exists k \in \{1,..,M\}$, $f_k(\vec{y}) < f_k(\vec{x})$.[1]

The solution of multi-objective optimization problems, is a set of non-dominated solutions called Pareto-Set (PS), which the corresponding set of these solutions in the objective space is called the Pareto-Front (PF).

In Multimodal Multi-Objective Optimization Problems, there are two or more distinct solutions in the PS, which correspond to the same value in the PF. In this area, most of the available literature deals with multimodal single objective optimization problemsand there is a relatively small number of published research on Multimodal Multi-objective Optimization Problems (MMOP).[2] In the current paper, we propose a new method for this type of problems, which is based on the combination of the Manhattan distance and the Crowding distance in decision space (MDCD).

The performance of our proposed method is examined on a number of available multimodal multi-objective test functions. We study the influence of the proposed method on finding a better approximation of optimal solutions in decision space, and the results are compared with the state-of-the-art algorithms.

The remaining parts of the paper proceed as follows: In section 2 the related works on MMOPs are investigated. The proposed algorithm is presented in section 3. In section 4, the setting of the experiments is explained. The experiments and analysis are presented in section 5. In the end, section 6 concludes the paper and provides the future research direction.

2 Related Work

In the field of Evolutionary Multi-Objective Optimization, the main concern is to find a good approximation of PF with a good diversity of solutions in objective space.[3] However, there There is not much literature focusing on increasing the diversity of solutions in the decision space to handle MMOPs.

One of the first works dealing with MMOPs was proposed by Deb and Tiwari[4] who introduce the omni-optimizer algorithm. This algorithm is a modified version of the well-known Non-Dominated-Genetic-Algorithm-II (NSGA-II).[5] The aim of this work was dealing with a wider range of optimization problems (i.e: single or multi-objective and uni or multi-modal problems). They proposed a modified crowding distance by comparing the crowding distance value of each individual with its average value (in both spaces), and take the larger value of the two distances.

To achieve a better distribution of solutions both in decision and objective spaces, Zhou et al.[6] proposed a model where the population is classified into sub-populations in the objective space, and the diversity of solutions is increased in the decision space by evaluating the diversity of PS in each sub-population. The obtained solutions show a better convergence to PS and PF for the MOP compared to the Omni-optimizer algorithm.

The concept of niching in MMOPs is used by Liang et al.[1] They proposed Decision-Based Niching NSGA-II (DN-NSGA-II) algorithm, where they applied the crowding distance technique to the decision space instead of the objective space as a secondary selection criteria. Even though this algorithm could find more Pareto optimal solutions than NSGA-II, the solutions are not well distributed in decision space.

Another perspective is found in an algorithm called *Multi-objective Particle Swarm Optimization using Ring topology by applying Special Crowding Distance* (MO-Ring-PSO-SCD) proposed by Yue et al.[7] They used a ring topology and a special crowding distance method to locate and maintain more Pareto optimal solutions. This algorithm is able to provide better approximation of PS in comparison with NSGA-II, DN-NSGA-II and Omni-optimizer algorithms.

Multimodal Multi-Objective Differential Evolution algorithm (MMODE) was proposed by Liang et al.[8] The mutation-bound process was introduced to provide a second opportunity to perform mutation for infeasible solutions (those outside the boundaries) of the decision space. In their presented algorithm, the crowding distance method is applied to the solutions in the decision space to maintain the diversity of solutions.

In a recent study, another contribution is proposed by Liu et al.,[2] called *Double-Niches Evolutionary Algorithm* (DNEA). The main focus of this method is the calculation of Euclidean distance in both decision and objective spaces. Then, a double-niched method is applied to diversify the solutions on both decision and objective spaces.

In a previous work, we proposed a modified version of NSGA-II algorithm called Weighted Sum Crowding Distance using NSGA-II algorithm (NSGA-II-WSCD).[9] To obtain a good diversity of solutions both in the decision and objective spaces, we compute the Crowding distance value of solutions by taking the weighted sum value of crowding distances in both spaces.

3 Proposed NSGA-II-MDCD Algorithm

In this section we propose a modified distance measurement technique that can be used to obtain a better diversity of solutions in decision space, and therefore make a better approximation of PS.

In the proposed method, due to the natural capability of grids to represent the distribution of solutions, we took the Manhattan distance metric (also called p_1 metric) as a distance measurement method in the decision space. For each solution, we calculate the Manhattan distance to all other solutions in the current front. Then, our global Manhattan Distance metric is computed as the summation of all of these distances between each solution and the rest of solutions. This metric is formulated as follows:

$$MD_{global}(\vec{a}) = \sum_{p \in P} \|\vec{a} - \vec{p}\| = \sum_{p \in P} \sum_{i=1}^{D} |a_i - pi| \qquad (2)$$

Where P is current front of solutions, D is the dimension of decision variables. Furthermore, a_i and p_i represent the grid index values of solutions $\vec{a}$ and $\vec{p}$ in dimension i.

In order to have a better diversity of solutions, we multiply the obtained global Manhattan distance metric value with its Crowding distance value in decision space (as defined in Omni-optimizer algorithm,[4] this distance only takes into account its nearest neighbor for boundaries). In Figure 1, we provide an example to better illustrate the influence of both Manhattan and Crowding distance on obtaining a good diversity of solutions in the decision space.

In Figure 1, the global Manhattan distance values of S_1 and S_2 are both equal to 20. Both of the solutions are located

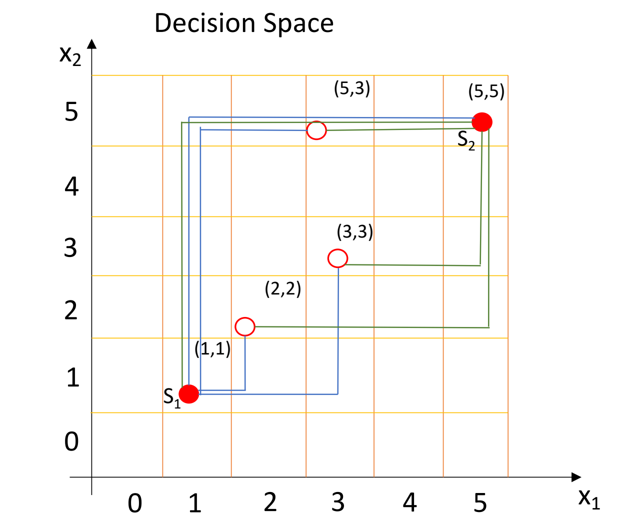


Figure 1: An example of the computation of MDCD, and its influence on the diversity of solutions in the decision space

far from the rest of the solutions and both make a good coverage of solutions in decision space. In this example, solution S_1 is located in a more crowded neighborhood area than S_2. Therefore, the crowding distance value for S_1 is smaller than for the other solutions. By multiplying both Manhattan and crowding distance values, S_2 gets a larger value than the other solutions. Therefore, we could guarantee a better diversity of solutions by using both distance metrics.

In algorithm 1 we present our proposed method (NSGA-II-MDCD) in details. We modify NSGA-II by changing the Crowding distance with our MDCD metric. First we calculate the global Manhattan distance value (Lines 1 to 7). Then, the crowding distance value for all solutions in the decision space is calculated (Lines 8 to 21). The Final *MDCD* value for each solution is calculated by multiplying the two distance values.

Algorithm 1: Combined Manhattan Distance and Crowding distance Approach (MDCD).

Input: Number of Objective functions: M,
Number of Decision Variables: D,
List P of solutions of current front (with GridIndex values for each dimension), of size $p = |P|$

Output: List P of solutions of current front with extra property of Combined Manhattan Distance and Crowding Distance (MDCD) for each solution

1 **for** $j \in \{1,..,p\}$ **do**
2     $P[j].MD_{global} = 0$
3     $P[j].CD_{dec} = 0$
4     $P[j].MDCD = 0$
5 **end**
6 **for** $i \in \{1,..,p\}$ **do**
7   **for** $j \in \{1,..,p\}$ **do**
8   **for** $k \in \{1,..,D\}$ **do**
9             $P[i].MD_{global} += P[i].GridIndex(k) - P[j].GridIndex(k))$
10   **end**
11   **end**
12 **end**
13 **for** $i \in \{1,..,D\}$ **do**
14     $x_{i,min}$ = minimum of values for i-th decision variable in P
15     $x_{i,max}$ = maximum of values for i-th decision variable in P
16 **end**
17 **for** $i \in \{1,..,D\}$ **do**
18   P' = sort P ascending based on i-th decision variable
19     $P'[1].CD_{dec} += 2 \cdot \frac{|P'[j+1].x_i - P'[j].x_i|}{|x_{i,max} - x_{i,min}|}$
20     $P'[p].CD_{dec} += 2 \cdot \frac{|P'[j].x_i - P'[j-1].x_i|}{|x_{i,max} - x_{i,min}|}$
21   **for** $j \in \{2,..,p-1\}$ **do**
22         $P'[j].CD_{dec} += \frac{|P'[j+1].x_i - P'[j-1].x_i|}{|x_{i,max} - x_{i,min}|}$
23   **end**
24 **end**
25 **for** $i \in \{1,..,p\}$ **do**
26     $P[j].MDCD = P[j].CD_{dec} \cdot P[j].MD_{sum}$
27 **end**
28 **return** P

4 Experiments

This section is dedicated to investigate the effectiveness of the proposed method (NSGA-II-MDCD) for obtaining a good approximation of solutions in both decision and objective spaces. We compare the proposed approach with the state-of-the-art, NSGA-II-WSCD algorithm and NSGA-II as a base-line algorithm.

4.1 Test Problems

We took 6 multimodal multi-objective test functions from the literature SSUF1, SSUF3[1] and MMF3-MMF6.[7] These test problems have different shapes and properties of the PF (concave and convex).

4.2 Parameter Settings

In the following we explain the parameter setting used in the comparisons. The population size is set to 100 and we used 10000 function evaluations as a termination criterion in all the experiments. We calculate the median and interquartile (IQR) ranges out of 31 independent runs. We used simulated Binary Crossover (SBX) and Polynomial Mutation as variation operators. The distribution index for both crossover and mutation is set to 20. The recombination probability $P_c = 1$ and the mutation probability $P_m = 1/D$. The grid size of the proposed algorithm is set to 10. We used the parameter value as in the literature where the WSCD value is obtained by equally division of weights in both decision and objective spaces. The implementation of these algorithms, as well as NSGA-II, is provided in the Matlab-based platform PlatEmo.[7]

Table 1: Featurs of Test Problems

Problem name	No. of Pareto subsets	PF Shape
SSUF1	2	concave
SSFU3	2	concave
MMF3	2	concave
MMF4	4	concave
MMF5	4	convex
MMF6	4	convex

4.3 Performance Measures

To assess the performance of the proposed method and the compared algorithms, we used the Inverted Generational Distance in decision space (IGDX).[6] The obtained values demonstrate both the diversity and convergence of solutions in decision space by calculating the Euclidean distance between the PS and the set of obtained solutions in decision space. The mathematical definition of IGDX is:

$$IGDX(P^*,R) = \frac{\sum_{v \in P^*} \|R - v\|_2}{|P^*|} \tag{3}$$

Where R and P^* accordingly are a set of obtained solutions in decision space and a sample of the PS, and $\|R-v\|_2$ is the minimum Euclidean distance between the sampled point v and any point in R.

We also look at the diversity and convergence of obtained solutions in objective space, by calculation of Inverted Generational distance (IGD),[10,11] which is mathematically formatted in the same way as IGDX as follows:

$$IGD(P^*,R) = \frac{\sum_{v \in P^*} \left\|\vec{f}(R) - \vec{f}(v)\right\|_2}{|P^*|} \tag{4}$$

Where R and P^* respectively are a set of obtained solutions in objective space and a sample of the PF.

5 Analysis of Results

The IGDX and IGD results are presented in Tables 2 and 3. The Mann-Whitney U statistical test is taken to test statistical significance according to the best algorithm on each test problems, and the significance is assumed for a value of $p \leq 0.05$. The values highlighted in bold represents the best values for each problem, and the asterisks (*) demonstrate the significance compared to the best algorithm for each test problem.

As can be observed in Table 2, NSGA-II-MDCD performs the best in terms of IGDX compared to the rest of the algorithms for four out of six test problems, which means that the proposed algorithm provides better distribution of solutions in the decision space. Even though NSGA-II-WSCD algorithm is getting better results for MMF3 and MMF4compared to the proposed method, no statistical significance was observed between these two algorithms. A possible explanation for this might be that by gridding the decision space, in MMF3 and MMF4, as the optimal solutions are more concentrated in concrete grids, then the NSGA-II-MDCD is getting worse results compared to other problems with optimal solutions involved in larger number of grids.

Table 2: IGDX value for comparison of different algorithms

	NSGA-II-MDCD	NSGA-II-WSCD	NSGA-II
SSUF1	**0.07478 (0.00849)**	0.07923 (0.00741)*	0.1051 (0.0151)*
SSFU3	**0.08699 (0.072)**	0.08949 (0.07273)	0.1021 (0.0853)
MMF3	0.07747 (0.03521)	**0.05839 (0.03499)**	0.07854(0.0314)
MMF4	0.06053 (0.01059)	**0.05793 (0.01098)**	0.11921 (0.04185)*
MMF5	**0.13723 (0.01042)**	0.14473 (0.01112)*	0.19475 (0.03932)*
MMF6	**0.11752 (0.00682)**	0.12406 (0.01261)*	0.18852 (0.06103)*

Table 3: IGD value for comparison of different algorithms

	NSGA-II-MDCD	NSGA-II-WSCD	NSGA-II
SSUF1	0.00662 (0.00053)*	0.00544 (0.00032)	**0.00532 (0.00026)**
SSFU3	0.02011 (0.02278)	0.01696 (0.0146)	**0.01995 (0.01253)**
MMF3	0.01805 (0.01474)	0.01527 (0.01329)	**0.0149 (0.00972)**
MMF4	0.00645 (0.00035)*	0.00542 (0.00025)*	**0.00517 (0.00019)**
MMF5	0.00655 (0.00034)*	0.00559 (0.00032)*	**0.00534 (0.00032)**
MMF6	0.00647 (0.0005)*	0.00549 (0.00028)*	**0.00531 (0.00026)**

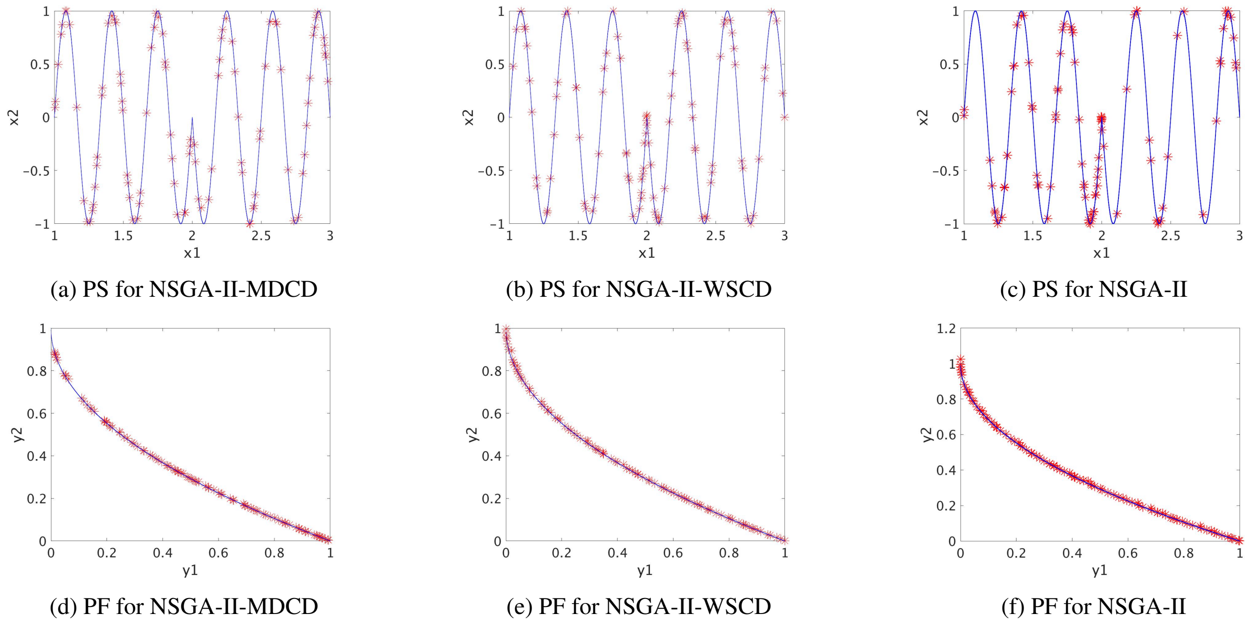
As we expected from Table 3, the IGD value of the NSGA-II algorithm shows its superiority in comparison with the proposed algorithm. The reason is that the main focus of NSGA-II algorithm is to get a better diversity of solutions in objective space, while neglecting decision space, therefore a lower IGD value is expected. According to the further analysis of results we cloud claim that NSGA-II-MDCD algorithm provides a better approximation of PS while not disturbing that much the approximation of PF.

In addition to these tables, we show the obtained solutions in both decision and objective spaces of the run with the median IGDX value for the SSUF1, MMF5 and MMF6 test problems in Figures 2, 3, and 4.

As can be seen from Figure 4 as an instance, the solutions of NSGA-II-MDCD are more evenly distributed in decision space than the solutions of NSGA-II-WSCD and NSGA-II algorithms. In objective space, the algorithm is still obtaining a good approximation of the PF, but some parts of it are less crowded than others in comparison with NSGA-II.

6 Conclusions

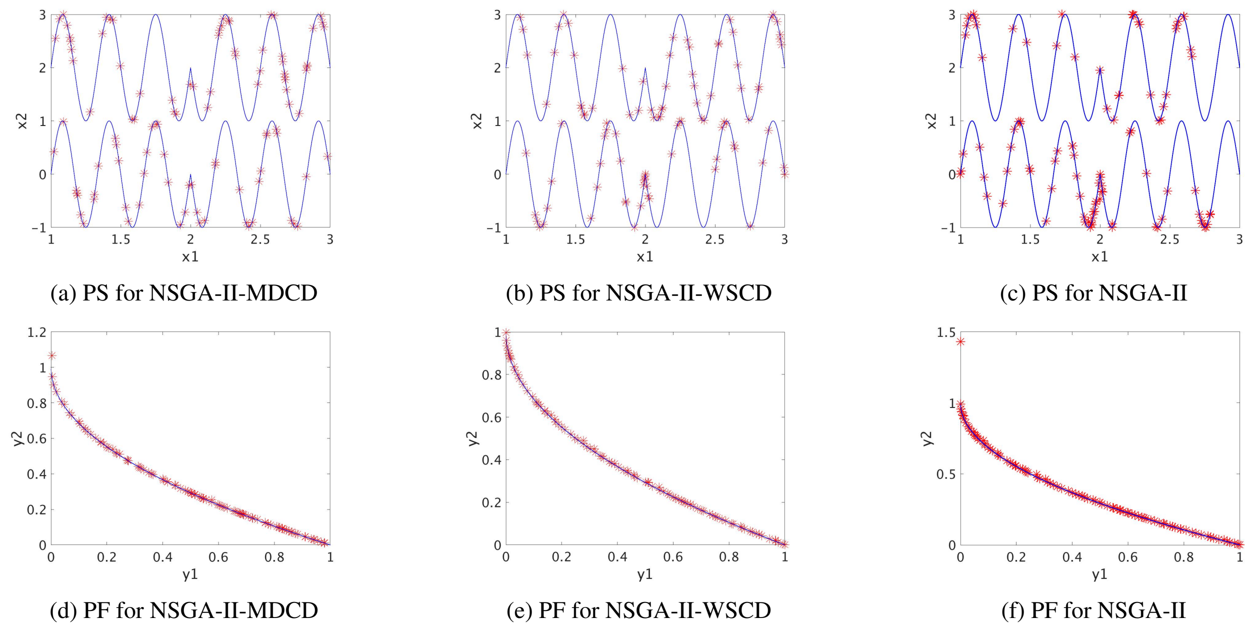
The goal of this study is to develop a method for MMOPs to provide a better approximation of solutions in the decision space. It is important to note that the good diversity of solutions in objective space does not guarantee a good diversity of solutions in decision space. As a result, we propose a technique to focus on increasing the distribution of solutions in decision space. We combine the Manhattan distance metric with crowding distance in decision space to satisfy our goal. Both distance measurement metrics together help to make a better distribution of solutions



(a) PS for NSGA-II-MDCD (b) PS for NSGA-II-WSCD (c) PS for NSGA-II

(d) PF for NSGA-II-MDCD (e) PF for NSGA-II-WSCD (f) PF for NSGA-II

Figure 2: Obtained solutions in decision and objective space for SSUF1 problem



(a) PS for NSGA-II-MDCD (b) PS for NSGA-II-WSCD (c) PS for NSGA-II

(d) PF for NSGA-II-MDCD (e) PF for NSGA-II-WSCD (f) PF for NSGA-II

Figure 3: Obtained solutions in decision and objective space for MMF5 problem

in decision space. The results of our experiments with 6 test problems show the superiority of the proposed method according to the approximation of PS over the NSGA-II-WSCD and NSGA-II algorithms.
Further studies are required to investigate the impact of the grid size on the quality of obtained optimal solutions in decision space. In addition, it is needed to develop techniques providing the ability of better local search to locate more optimal solutions.

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