Inference in Belief Trees

Inference in Belief Networks

A Bayesian Network is a complete model for the variables and their relationships.

It can be used to answer queries about them.

Typical question:
Given observes variables, what is the updated knowledge about the other variables.

There are exact inference methods for this task.

Choice of universe of discourse

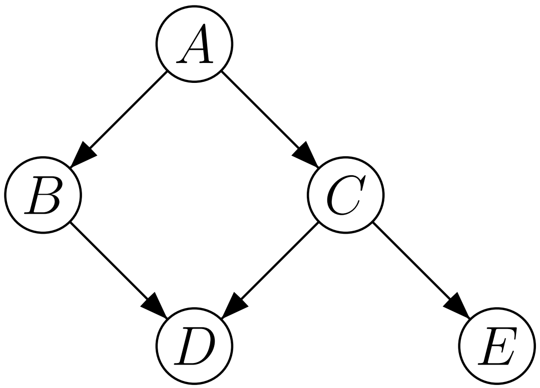
	Variable	Domain
A	metastatic cancer	$\{a_1, a_2\}$
B	increased serum calcium	$\{b_1, b_2\}$
C	brain tumor	$\{c_1, c_2\}$
D	coma	$\{d_1, d_2\}$
E	headache	$\{e_1, e_2\}$

($\cdot_1$ — present, $\cdot_2$ — absent)

$$\Omega = \{a_1, a_2\} \times \cdots \times \{e_1, e_2\}$$

$$|\Omega| = 32$$

Analysis of dependencies



Motivation

$$\left.\begin{aligned} P(e_1 \mid c_1) &= 0.8 \\ P(e_1 \mid c_2) &= 0.6 \end{aligned}\right\} \text{headaches common, but more common if tumor present}$$

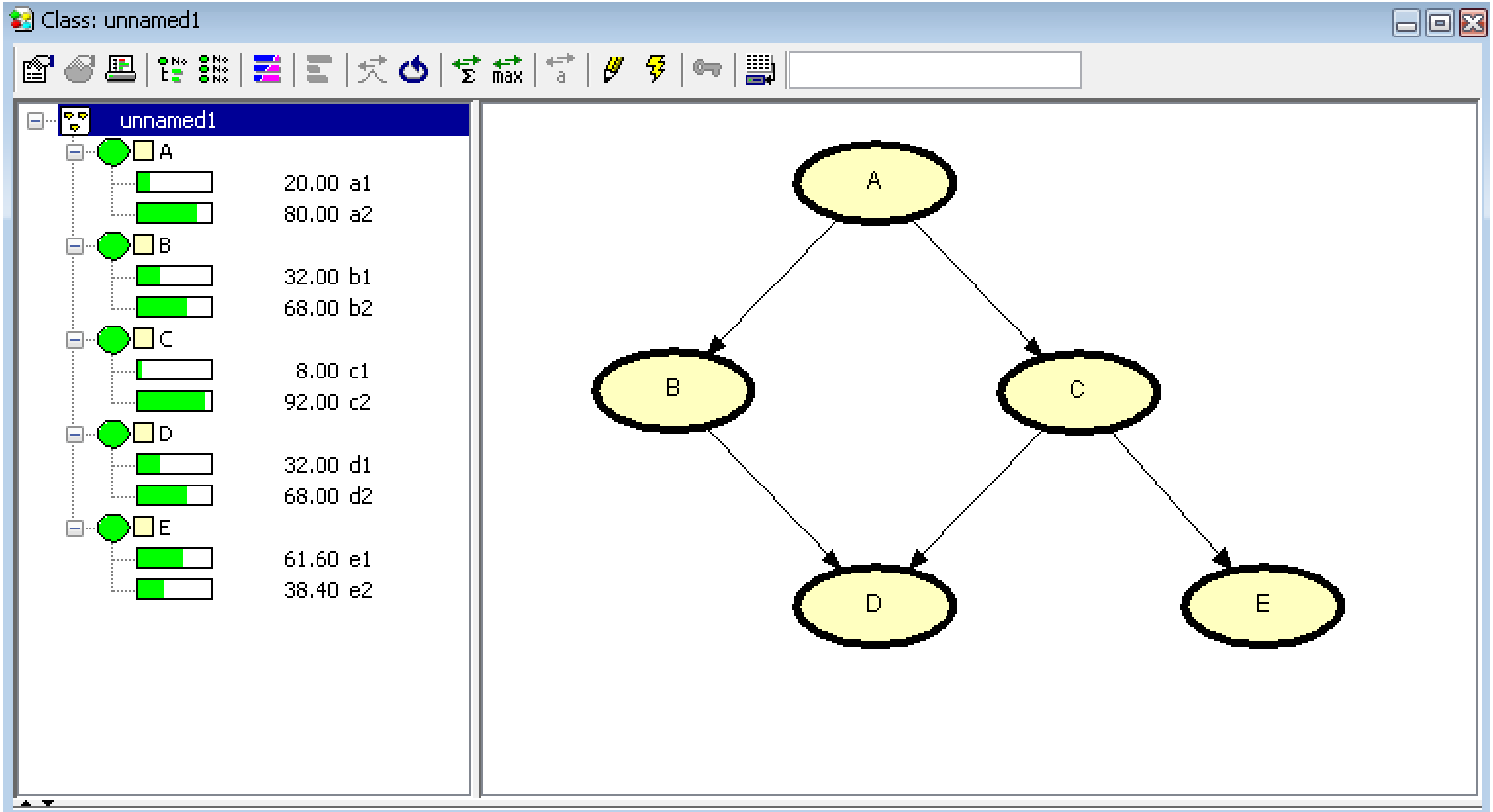
$$\left.\begin{aligned} P(d_1 \mid b_1, c_1) &= 0.8 \\ P(d_1 \mid b_1, c_2) &= 0.8 \\ P(d_1 \mid b_2, c_1) &= 0.8 \\ P(d_1 \mid b_2, c_2) &= 0.05 \end{aligned}\right\} \text{coma rare but common, if either cause is present}$$

$$\left.\begin{aligned} P(b_1 \mid a_1) &= 0.8 \\ P(b_1 \mid a_2) &= 0.2 \end{aligned}\right\} \begin{array}{c}\text{increased calcium uncommon,} \\ \text{but common consequence of metastases}\end{array}$$

$$\left.\begin{aligned} P(c_1 \mid a_1) &= 0.2 \\ P(c_1 \mid a_2) &= 0.05 \end{aligned}\right\} \text{brain tumor rare, and uncommon consequence of metastases}$$

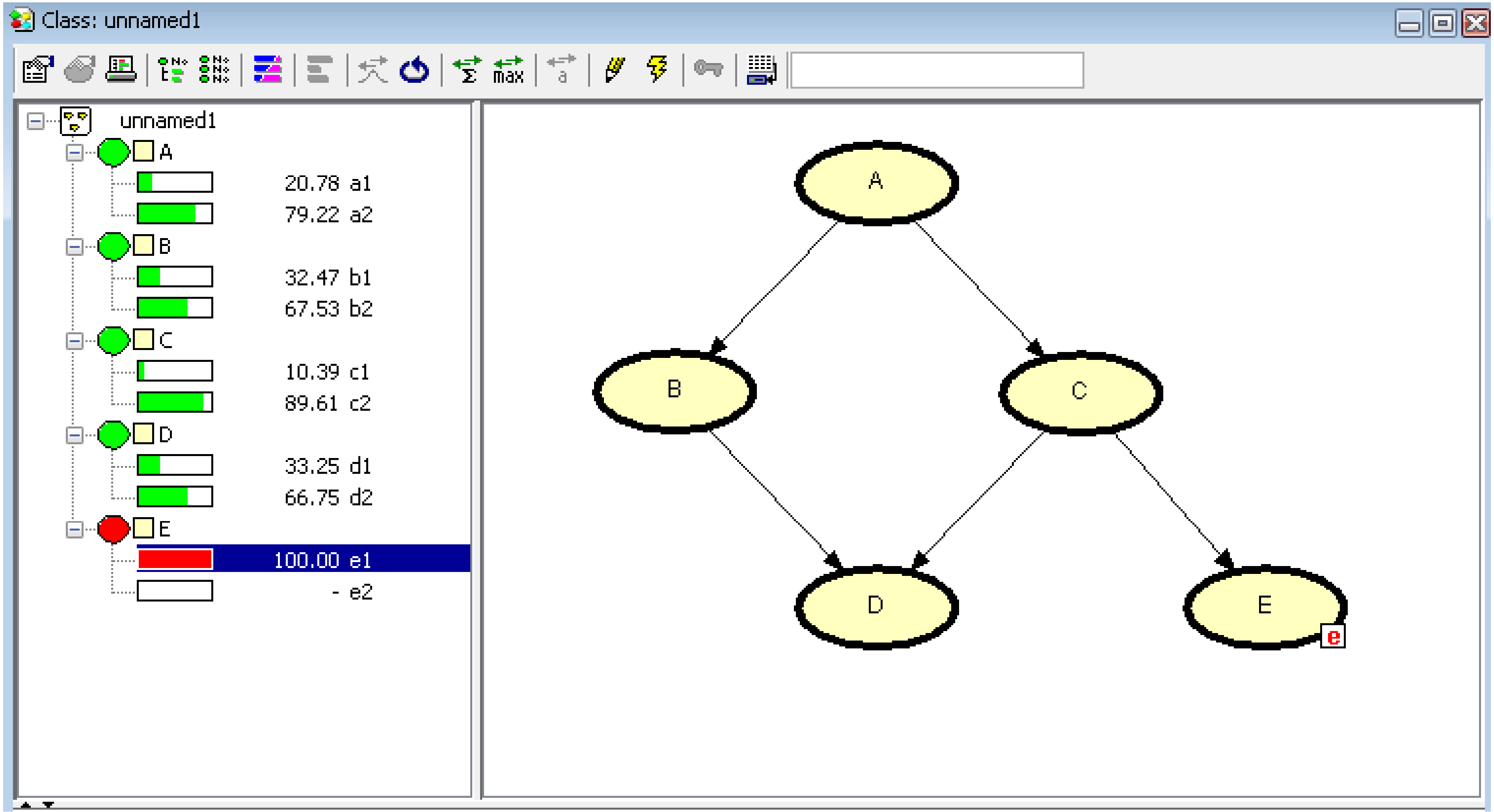
$$\left.P(a_1) = 0.2 \right\} \text{incidence of metastatic cancer in relevant clinic}$$

Motivation



Marginal distributions in the HUGIN tool.

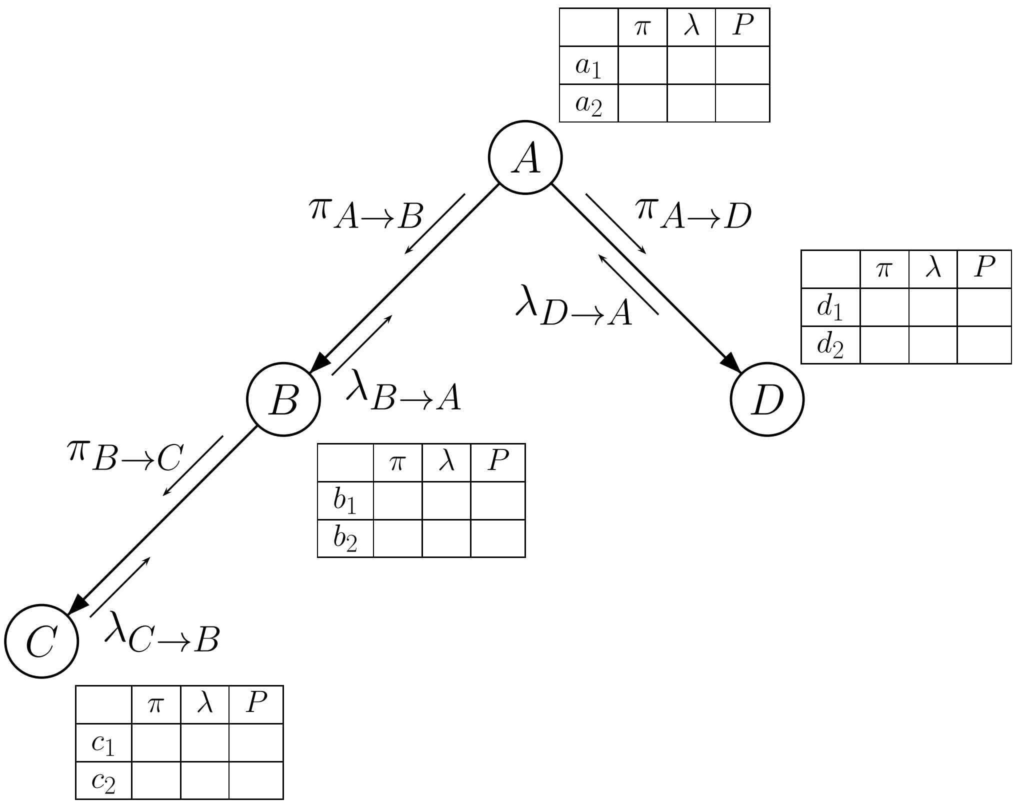
Motivation



Conditional marginal distributions with evidence $E = e_1$

Propagation in Belief Trees

Belief Tree:



Parameters:

$$P(a_1) = 0.1 \quad P(b_1 \mid a_1) = 0.7$$
$$P(b_1 \mid a_2) = 0.2$$
$$P(d_1 \mid a_1) = 0.8 \quad P(c_1 \mid b_1) = 0.4$$
$$P(d_1 \mid a_2) = 0.4 \quad P(c_1 \mid b_2) = 0.001$$

Desired:

$$P(X \mid Y)$$

Objective

Given: Belief network (V, E, P) with tree structure.
Let W be the set of variables for which the true value is known, i.e. the instantiated variables

Desired: $P(B \mid W)$ for all $B \in V$

Notation:

W_B^- subset of those variables of W that belong to the subtree of (V, E) that has root B

W_B^+ $= W \backslash W_B^-$

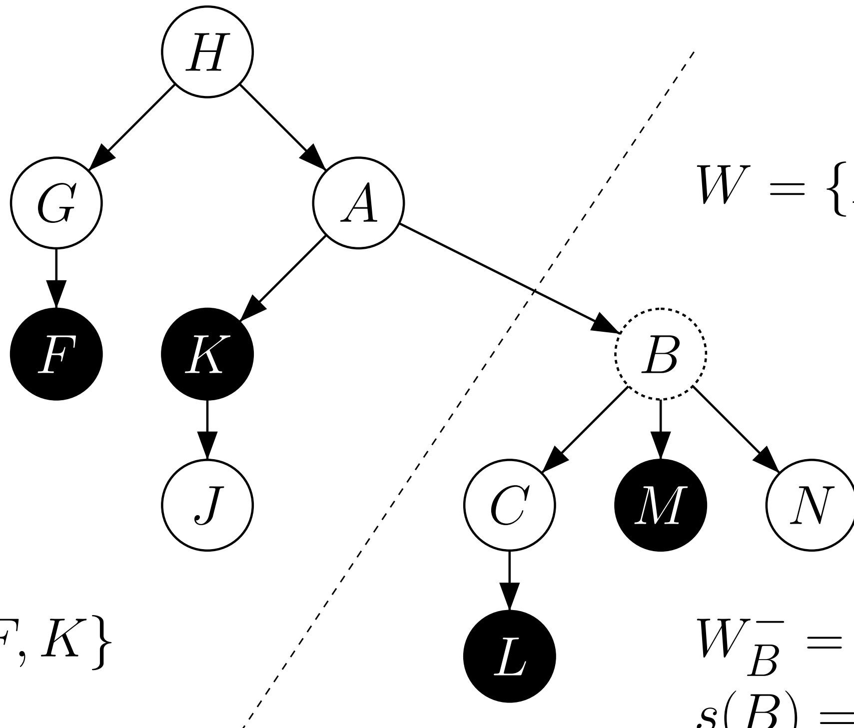
$s(B)$ set of direct successors of B

Ω_B domain of B

If a variable X is instantiated, then this value is denoted by x^*

$\alpha,\ \beta,\ \rho,\ \tau$ are normalizing factors

Example



$$W = \{F, K, L, M\}$$

$$W_B^+ = \{F, K\}$$

$$W_B^- = \{L, M\}$$

$$s(B) = \{C, M, N\}$$

Decomposition

$$
\begin{aligned}
P(B = b \mid W) &= P(b \mid W_B^- \cup W_B^+) \qquad \text{with } B \notin W \\
&= \frac{P(W_B^- \cup W_B^+ \cup \{b\})}{P(W_B^- \cup W_B^+)} \\
&= \frac{P(W_B^- \cup W_B^+ \mid b) P(b)}{P(W_B^- \cup W_B^+)} \\
&= \frac{P(W_B^- \mid b) P(W_B^+ \mid b) P(b)}{P(W_B^- \cup W_B^+)} \\
&= \beta_{B,W} \quad \underbrace{P(W_B^- \mid b)}_{\text{Evidence from ``below''}} \quad \underbrace{P(b \mid W_B^+)}_{\text{Evidence from ``above''}}
\end{aligned}
$$

π-values and λ-values

Let $B \in V$ be a variable and $b \in \Omega_B$ a value of its domain. We define the π- and λ-values as follows:

$$\lambda(b) = \begin{cases} P(W_B^- \mid b) & \text{if } B \notin W \\ 1 & \text{if } B \in W \wedge b^* = b \\ 0 & \text{if } B \in W \wedge b^* \neq b \end{cases}$$

$$\pi(b) = P(b \mid W_B^+)$$

π- and λ-Values

$$\lambda(b) = \prod_{C \in s(B)} P(W_C^- \mid b) \qquad \text{if } B \notin W$$

$$\lambda(b) = 1 \qquad \text{if } B \text{ leaf in } (V, E)$$

$$\pi(b) = P(b) \qquad \text{if } B \text{ root in } (V, E)$$

$$P(b \mid W) = \alpha_{B,W} \cdot \lambda(b) \cdot \pi(b)$$

λ-message

Let $B \in V$ be an attribute and $C \in s(B)$ its direct children with the respective domains $\text{dom}(B) = \{B_1, \dots, b_i, \dots, b_k\}$ and $\text{dom}(C) = \{c_1, \dots, c_j, \dots, c_m\}$.

$$\lambda_{C \to B}(b_i) \overset{\text{Def}}{=} \sum_{j=1}^{m} P(c_j \mid b_i) \cdot \lambda(c_j), \qquad i = 1, \dots, k$$

The vector

$$\vec{\lambda}_{C \to B} \overset{\text{Def}}{=} \Big(\lambda_{C \to B}(b_i)\Big)_{i=1}^{k}$$

is called λ-*message* from C to B.

λ-Message

Let $B \in V$ an attribute an $b \in \mathrm{dom}(B)$ a value of its domain.
Then

$$\lambda(b) = \begin{cases} \rho_{B,W} \cdot \prod\limits_{C \in s(B)} \lambda_{C \to B}(b) & \text{if } B \notin W \\ 1 & \text{if } B \in W \wedge b = b^* \\ 0 & \text{if } B \in W \wedge b \neq b^* \end{cases}$$

with $\rho_{B,W}$ being a normalizing factor

π-message

Let $B \in V$ be a non-root node in (V, E) and $A \in V$ its parent with domain $\mathrm{dom}(A) = \{a_1, \dots, a_j, \dots, a_m\}$.

$$j = 1, \dots, m :$$

$$\pi_{A \to B}(a_j) \overset{\text{Def}}{=} \begin{cases} \pi(a_j) \cdot \displaystyle\prod_{C \in s(A) \setminus \{B\}} \lambda_{C \to B}(a_j) & \text{if } A \notin W \\ 1 & \text{if } A \in W \wedge a = a^* \\ 0 & \text{if } A \in W \wedge a \neq a^* \end{cases}$$

The vector

$$\vec{\pi}_{A \to B} \overset{\text{Def}}{=} \Big(\pi_{A \to B}(a_j)\Big)_{j=1}^{m}$$

is called *π-message* from A to B.

π-Message

Let $B \in V$ be a non-root node in (V, E) and A the parent node of B.
Further let $b \in \text{dom}(B)$ be a value of B's domain.

$$\pi(b) \;=\; \mu_{B,W} \;\cdot \sum_{a \in \text{dom}(A)} P(b \mid a) \cdot \pi_{A \to B}(a)$$

Let $A \notin W$ a non-instantiated attribute and $P(V) > 0$.

$$\pi_{A \to B}(a_j) \;=\; \pi(a_j) \;\cdot \prod_{C \in s(A) \setminus \{B\}} \lambda_{C \to A}(a_j)$$

$$= \; \tau_{B,W} \cdot \frac{P(a_j \mid W)}{\lambda_{B \to A}(a_j)}$$

with $\tau_{B,W}$ being a normalizing factor

Messages - if B is instantiated

1. Set

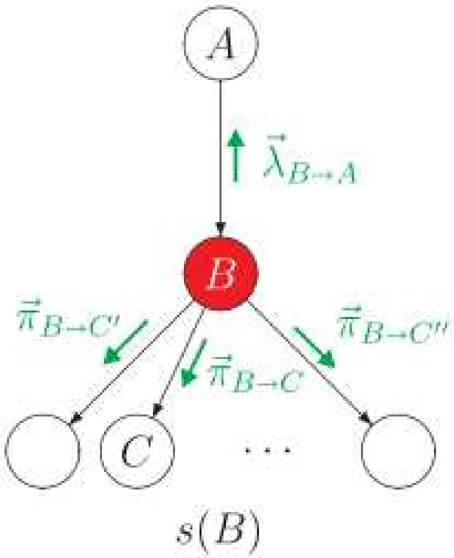
$$\lambda(b) = P(b \mid W) = \begin{cases} 1 & \text{if } B \in W \wedge b = b^* \\ 0 & \text{if } B \in W \wedge b \neq b^* \end{cases}$$

2. Post new $\lambda-$message $\lambda_{B\to A}$ to all parents of B

$$\lambda_{B\to A}(a) = \sum_{b\in\Omega_B} P(b|a) \cdot \lambda(b)$$

3. Post new $\pi-$message $\pi_{B\to C}$ to all successors of B.

$$\pi_{B\to C}(b) = \begin{cases} 1 & \text{if } B \in W \wedge b = b^* \\ 0 & \text{if } B \in W \wedge b \neq b^* \end{cases}$$



Messages - if B is not instantiated, and B received $\lambda_{C \to B}$

1. Compute new $\lambda-$values $\forall b \in \Omega_B$:

$$\lambda(b) = \rho_{B,W} \cdot \prod_{C \in s(B)} \lambda_{C \to B}(b) \qquad \text{if } B \notin W$$

2. Compute new $P(b \mid W)$ values $\forall b \in \Omega_B$:

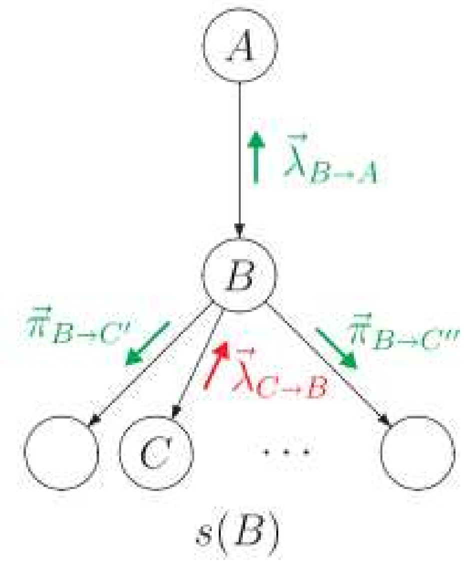
$$P(b \mid W) = \alpha_{B,W} \cdot \pi(b) \cdot \lambda(b)$$

3. Post new $\lambda-$message $\lambda_{B \to A}$ to all parents of B:

$$\lambda_{B \to A}(a) = \sum_{b \in \Omega_B} P(b|a) \cdot \lambda(b)$$

4. Post new $\pi-$message $\lambda_{B \to C}$ to all successors of B. $\forall b \in \Omega_B$:

$$\pi_{B \to C}(b) = \tau_{B,W} \frac{P(b \mid W)}{\lambda_{C \to B}(b)}$$



Messages - if B is not instantiated, and B received $\pi_{A\to B}$

1. Compute new $\pi-$values $\forall b \in \Omega_B$:

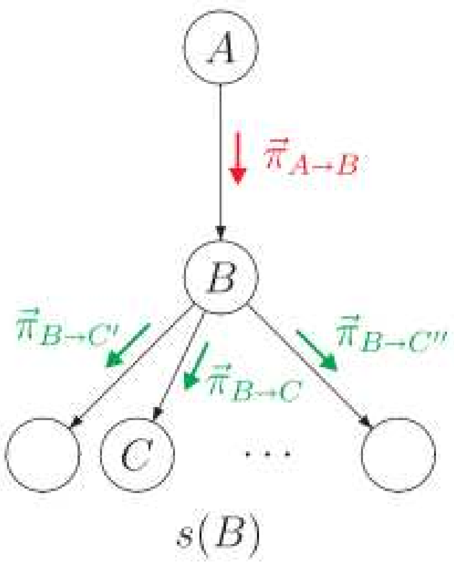
$$\pi(b) = \mu_{B,W} \cdot \sum_{b\in\Omega_B} P(b \mid a) \cdot \pi_{A\to B} \qquad \text{if } B \notin W$$

2. Compute new $P(b \mid W)$ values

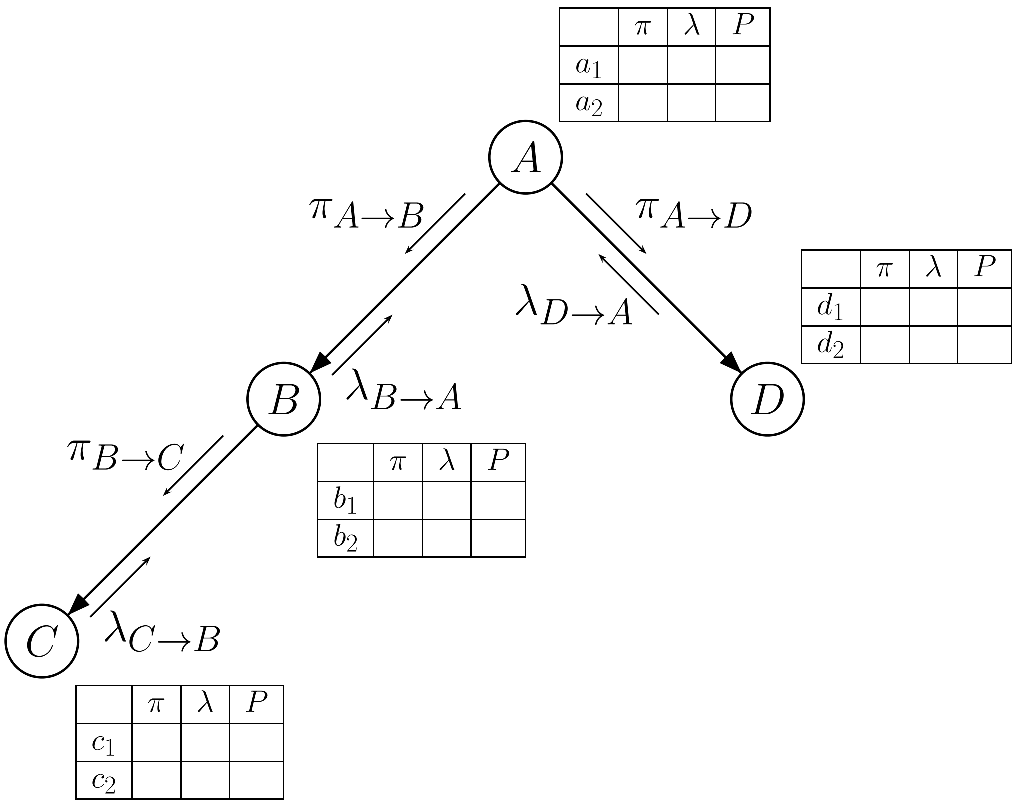
$$P(b \mid W) = \alpha_{B,W} \cdot \pi(b) \cdot \lambda(b)$$

3. Post new $\pi-$message $\lambda_{B\to C}$ to all successors of B.

$$\pi_{B\to C}(b) = \tau_{B,W} \cdot \frac{P(b \mid W)}{\lambda_{C\to B}(b)}$$



Belief Tree:



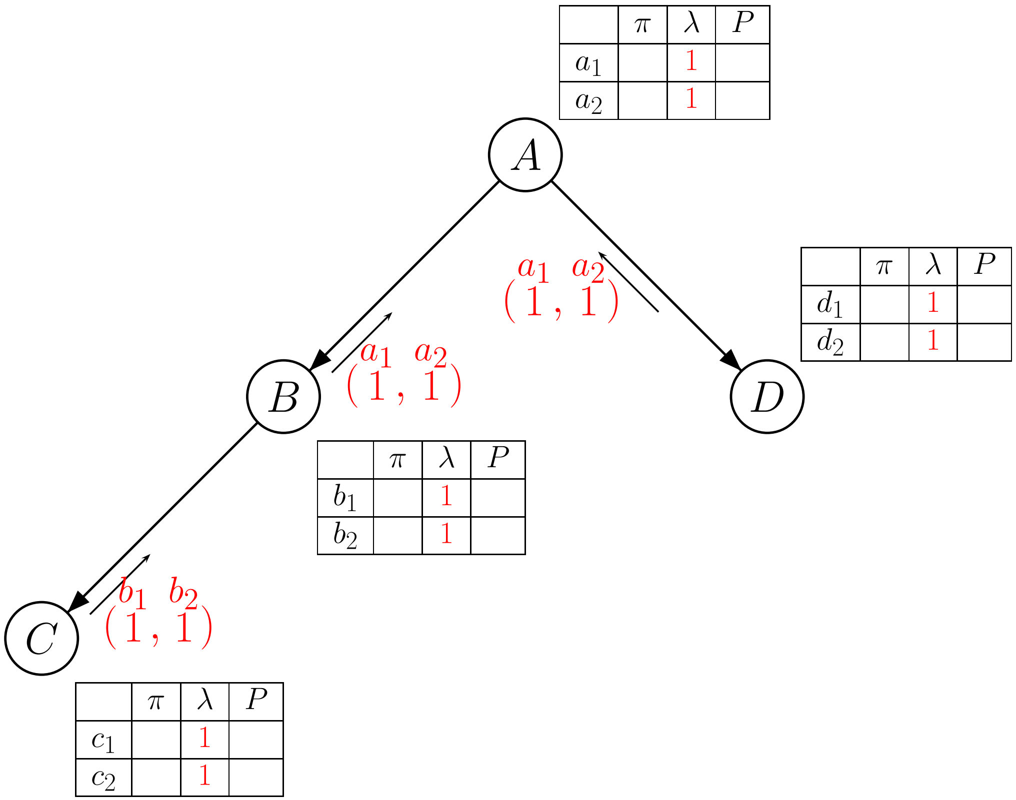
Parameters:

$$P(a_1) = 0.1 \quad P(b_1 \mid a_1) = 0.7$$
$$P(b_1 \mid a_2) = 0.2$$
$$P(d_1 \mid a_1) = 0.8 \quad P(c_1 \mid b_1) = 0.4$$
$$P(d_1 \mid a_2) = 0.4 \quad P(c_1 \mid b_2) = 0.001$$

Desired:

$$P(A) \quad P(B) \quad P(C) \quad P(D)$$

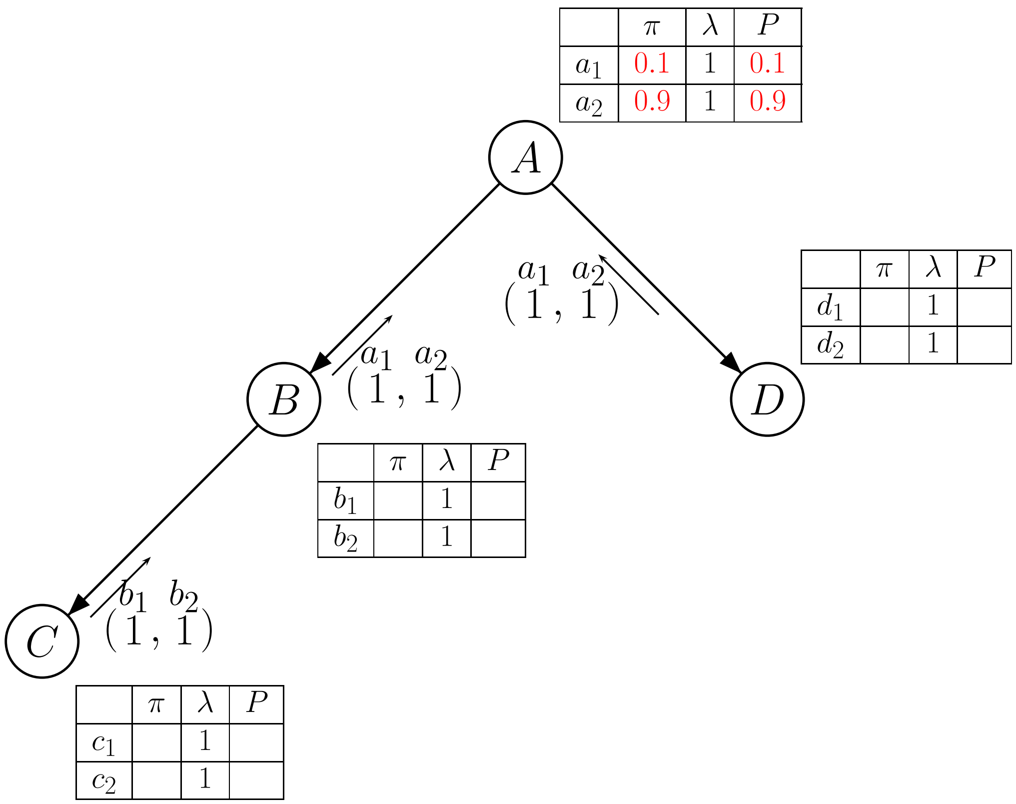
Belief Tree:



Initialization Phase:

Set all λ-messages and λ-values to 1.

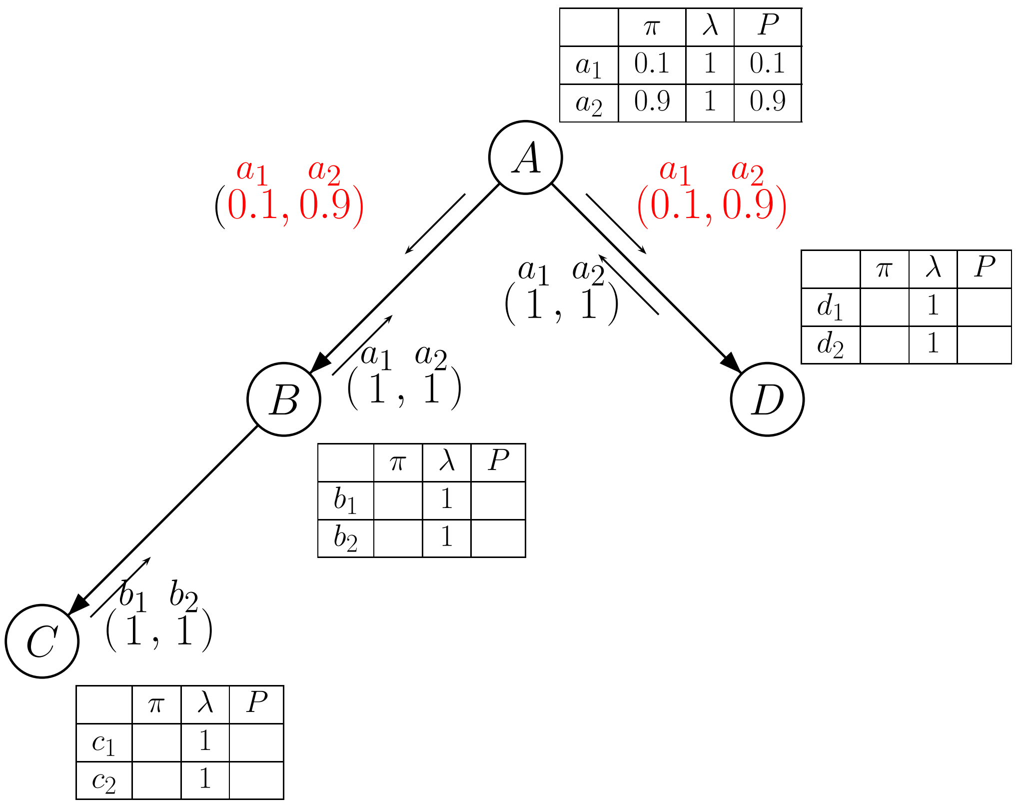
Belief Tree:



Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and
$\pi(a_2) = P(a_2)$

Belief Tree:

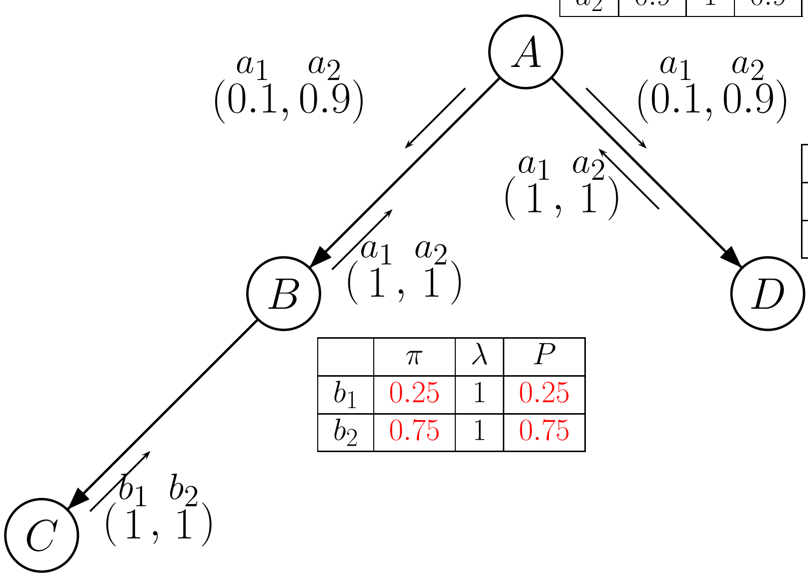


Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.

Propagation in Belief Trees (5)

Belief Tree:



	π	λ	P
a_1	0.1	1	0.1
a_2	0.9	1	0.9

	π	λ	P
d_1	0.44	1	0.44
d_2	0.56	1	0.56

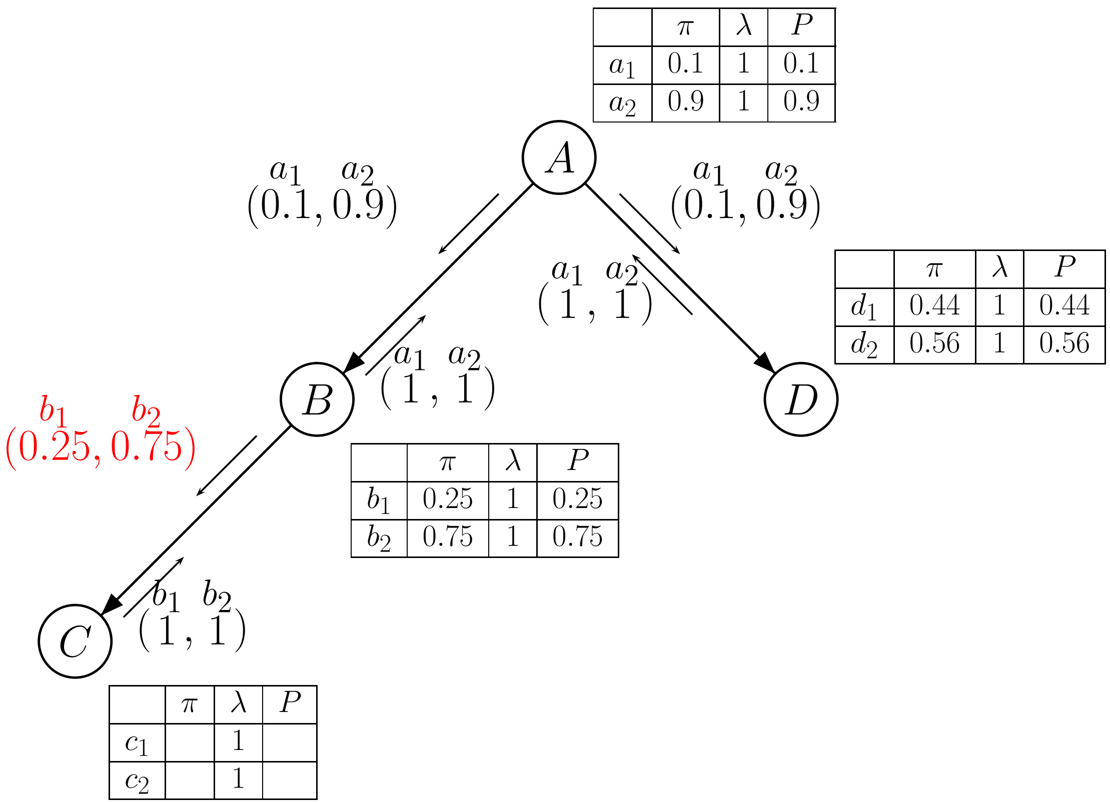
	π	λ	P
b_1	0.25	1	0.25
b_2	0.75	1	0.75

	π	λ	P
c_1		1	
c_2		1	

Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.
B and D update their π-values.

Belief Tree:



	π	λ	P
a_1	0.1	1	0.1
a_2	0.9	1	0.9

	π	λ	P
d_1	0.44	1	0.44
d_2	0.56	1	0.56

	π	λ	P
b_1	0.25	1	0.25
b_2	0.75	1	0.75

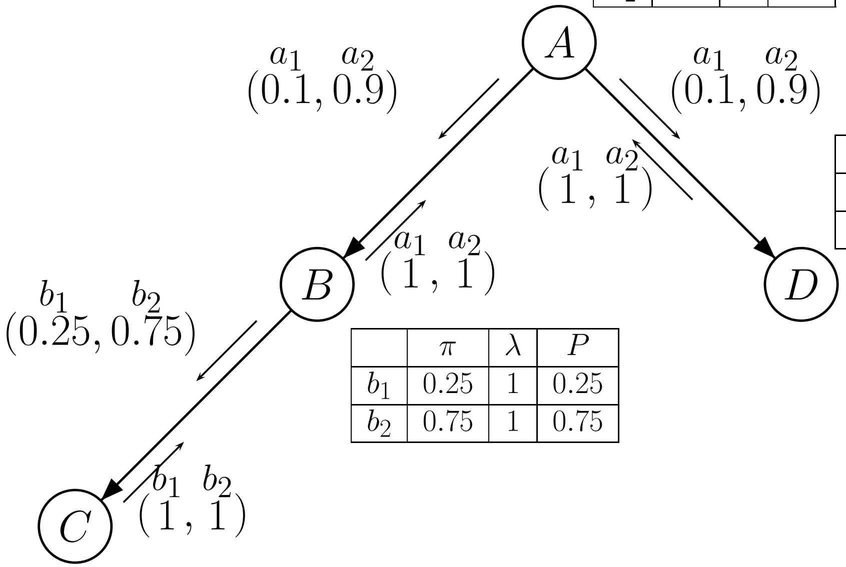
	π	λ	P
c_1		1	
c_2		1	

Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.
B and D update their π-values.
B sends π-message to C.

Propagation in Belief Trees (7)

Belief Tree:



	π	λ	P
a_1	0.1	1	0.1
a_2	0.9	1	0.9

	π	λ	P
d_1	0.44	1	0.44
d_2	0.56	1	0.56

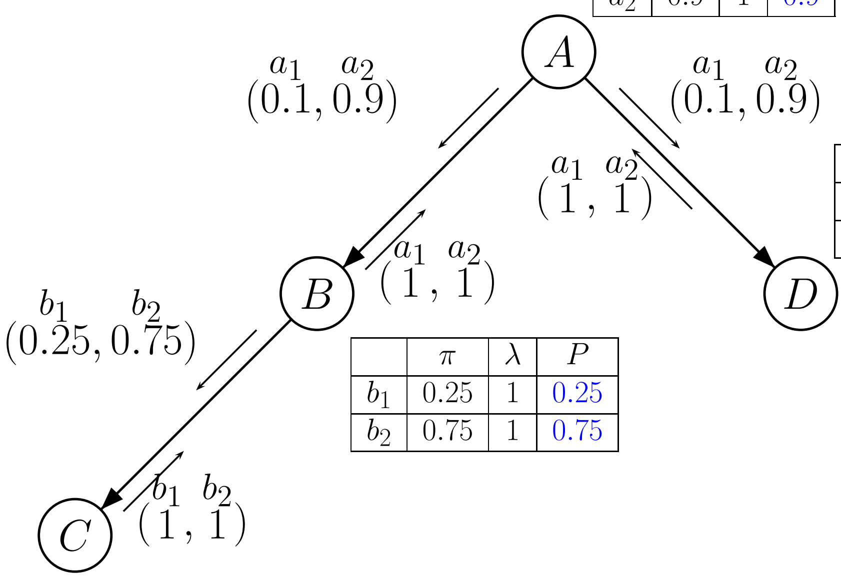
	π	λ	P
b_1	0.25	1	0.25
b_2	0.75	1	0.75

	π	λ	P
c_1	0.10075	1	0.10075
c_2	0.89925	1	0.89925

Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.
B and D update their π-values.
B sends π-message to C.
C updates it π-value.

Belief Tree:



	π	λ	P
a_1	0.1	1	0.1
a_2	0.9	1	0.9

	π	λ	P
d_1	0.44	1	0.44
d_2	0.56	1	0.56

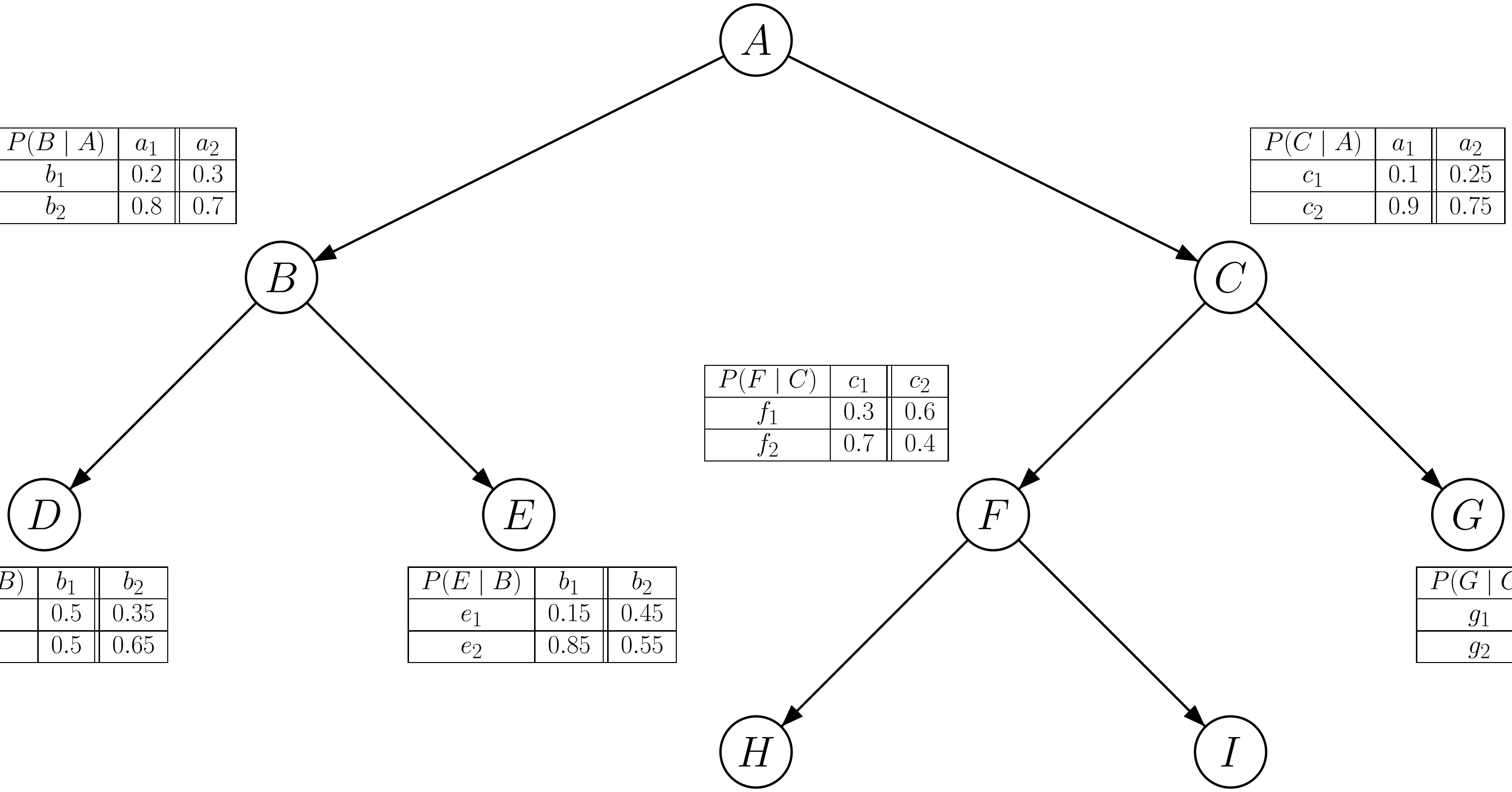
	π	λ	P
b_1	0.25	1	0.25
b_2	0.75	1	0.75

	π	λ	P
c_1	0.10075	1	0.10075
c_2	0.89925	1	0.89925

Initialization Phase:

Set all λ-messages and λ-values to 1.
$\pi(a_1) = P(a_1)$ and $\pi(a_2) = P(a_2)$.
A sends π-messages to B and D.
B and D update their π-values.
B sends π-message to C.
C updates it π-value.
Initialization finished.

Larger Network (1): Parameters



$P(A)$	$\emptyset$
a_1	0.4
a_2	0.6

$P(B \mid A)$	a_1	a_2
b_1	0.2	0.3
b_2	0.8	0.7

$P(C \mid A)$	a_1	a_2
c_1	0.1	0.25
c_2	0.9	0.75

$P(F \mid C)$	c_1	c_2
f_1	0.3	0.6
f_2	0.7	0.4

$P(D \mid B)$	b_1	b_2
d_1	0.5	0.35
d_2	0.5	0.65

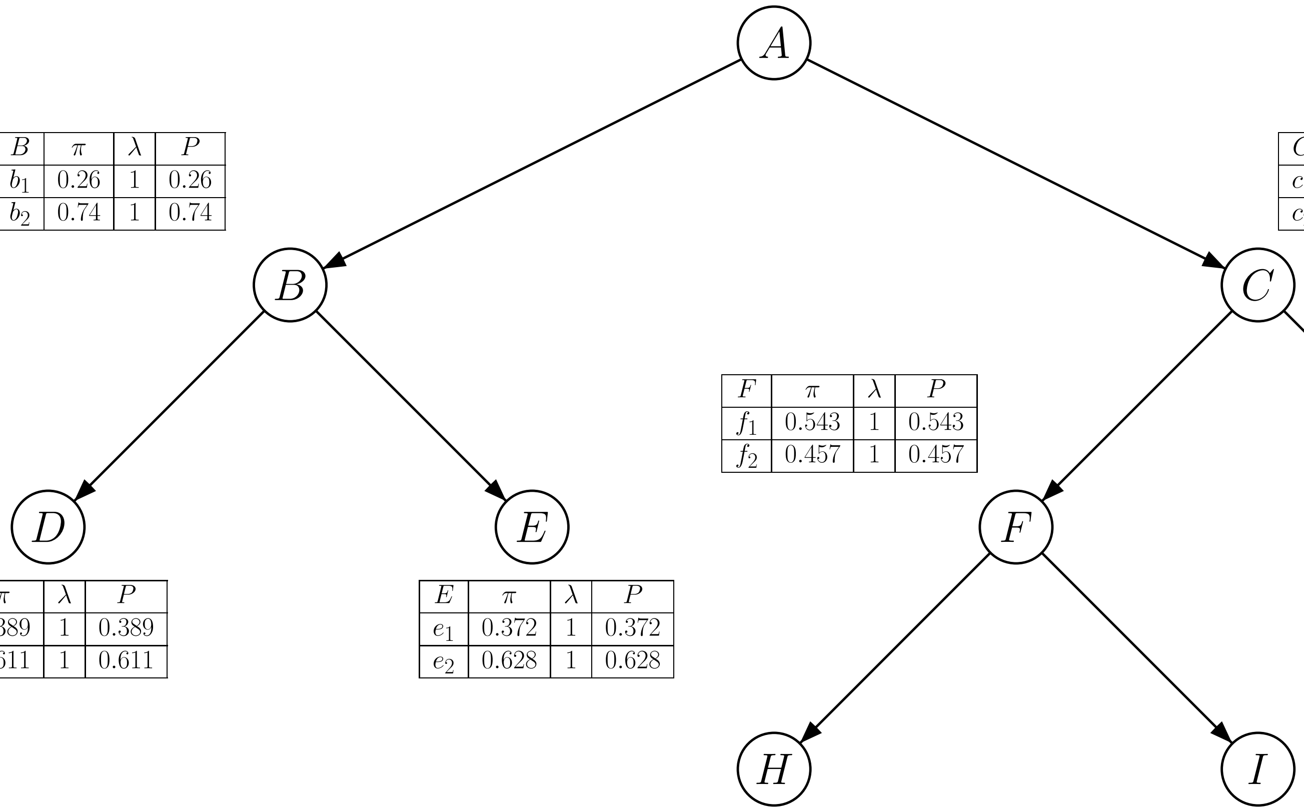
$P(E \mid B)$	b_1	b_2
e_1	0.15	0.45
e_2	0.85	0.55

$P(G \mid C)$	c_1	c_2
g_1	0.25	0.1
g_2	0.75	0.9

$P(H \mid F)$	f_1	f_2
h_1	0.65	0.2
h_2	0.35	0.8

$P(I \mid F)$	f_1	f_2
i_1	0.25	0.5
i_2	0.75	0.5

Larger Network (2): After Initialization



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	1	0.26
b_2	0.74	1	0.74

C	π	λ	P
c_1	0.19	1	0.19
c_2	0.81	1	0.81

F	π	λ	P
f_1	0.543	1	0.543
f_2	0.457	1	0.457

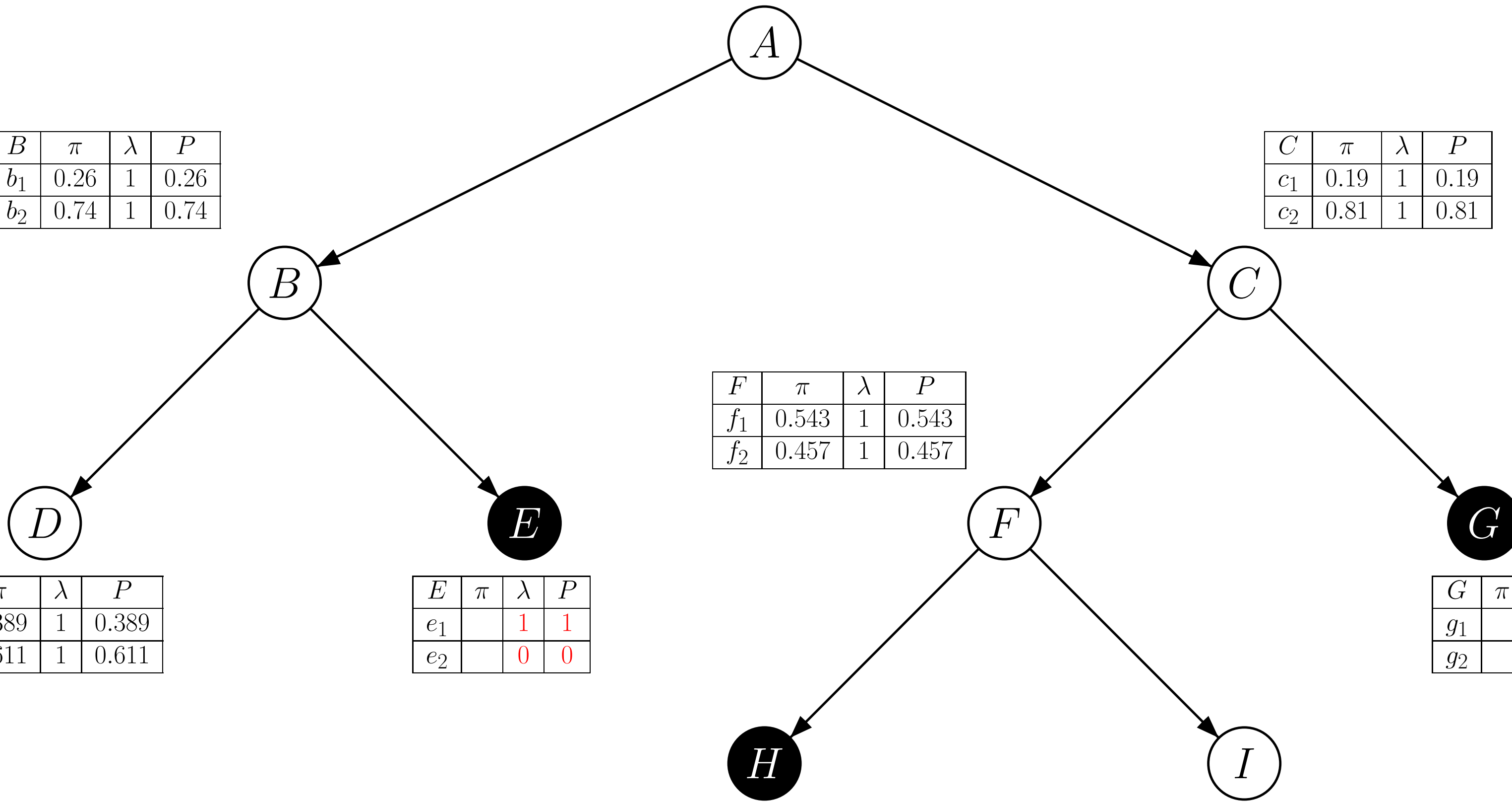
D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

E	π	λ	P
e_1	0.372	1	0.372
e_2	0.628	1	0.628

G	π	λ	P
g_1	0.1285	1	0.1285
g_2	0.8715	1	0.8715

H	π	λ	P
h_1	0.4444	1	0.4444
h_2	0.5556	1	0.5556

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	1	0.26
b_2	0.74	1	0.74

C	π	λ	P
c_1	0.19	1	0.19
c_2	0.81	1	0.81

F	π	λ	P
f_1	0.543	1	0.543
f_2	0.457	1	0.457

D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

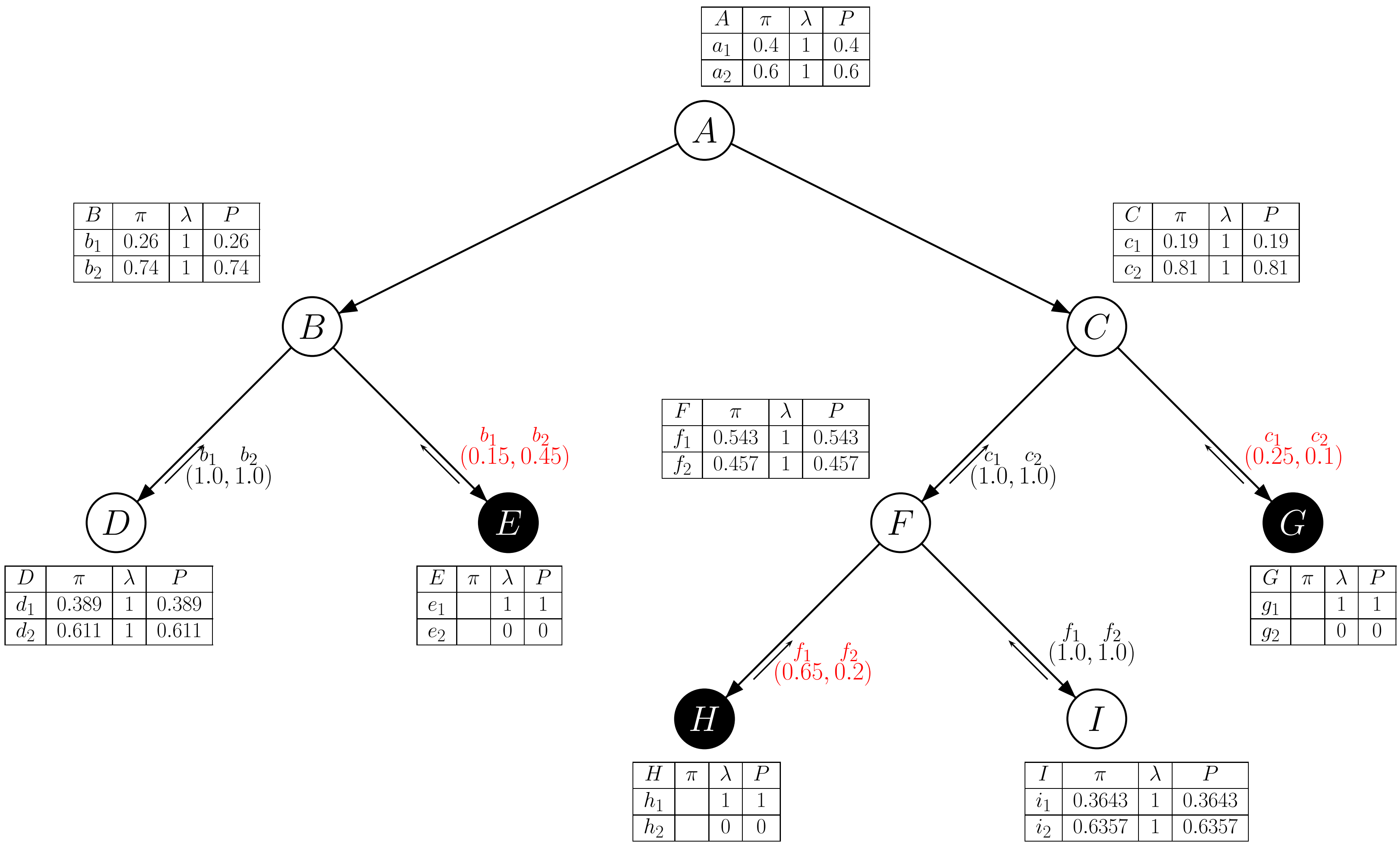
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357

Larger Network (4): Propagate Evidence



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	1	0.26
b_2	0.74	1	0.74

C	π	λ	P
c_1	0.19	1	0.19
c_2	0.81	1	0.81

F	π	λ	P
f_1	0.543	1	0.543
f_2	0.457	1	0.457

D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

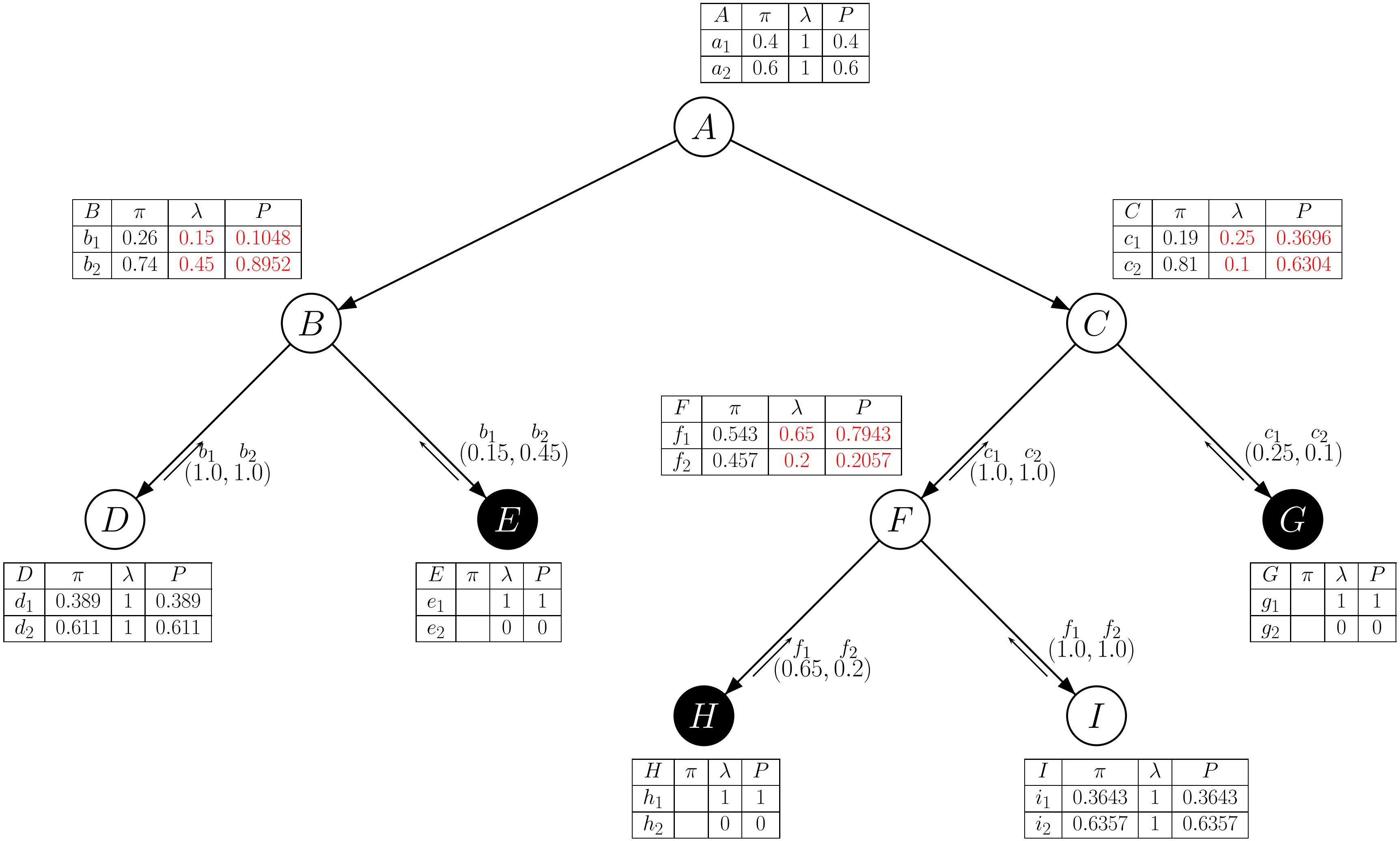
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357

Larger Network (5): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.25	0.3696
c_2	0.81	0.1	0.6304

F	π	λ	P
f_1	0.543	0.65	0.7943
f_2	0.457	0.2	0.2057

D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

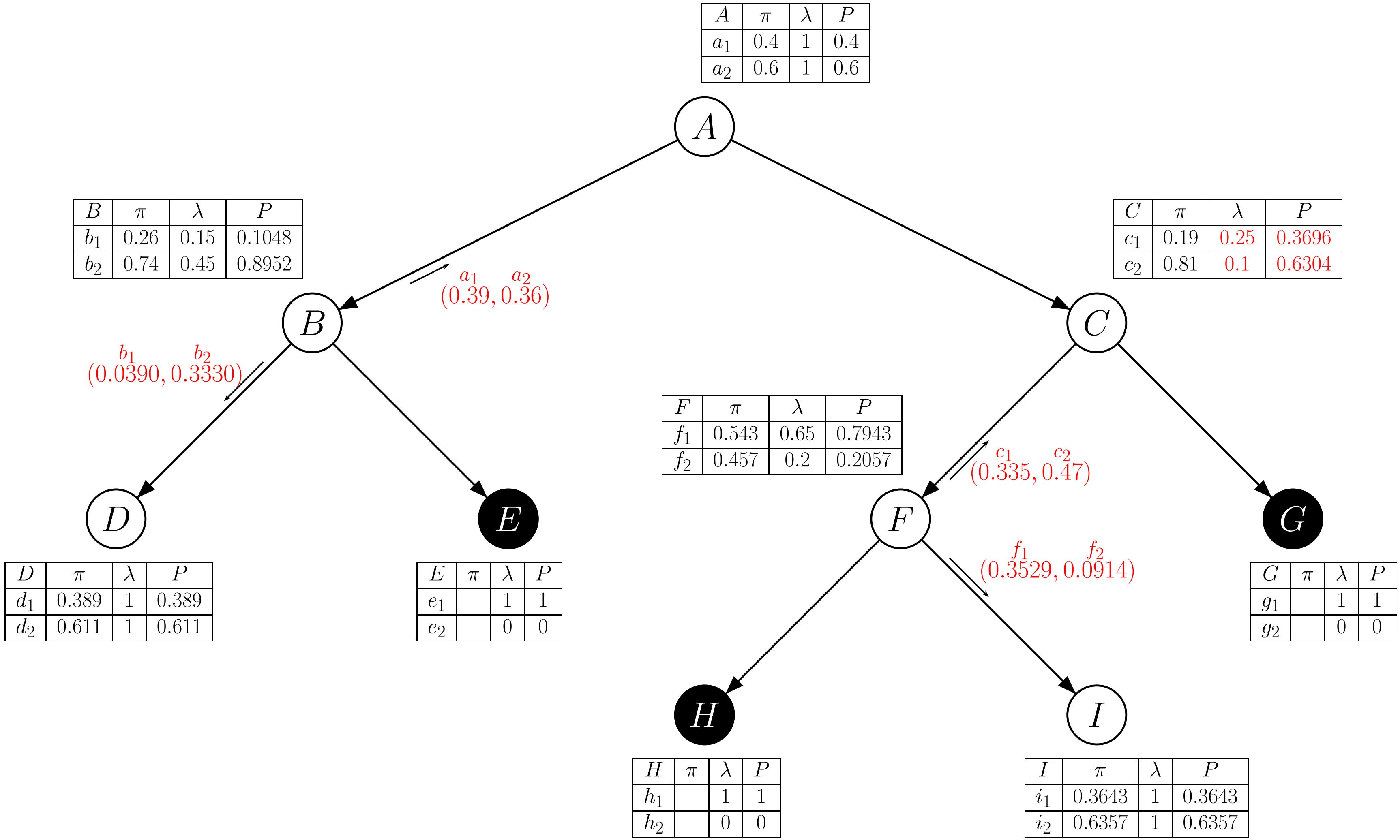
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357

Larger Network (6): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	1	0.4
a_2	0.6	1	0.6

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.25	0.3696
c_2	0.81	0.1	0.6304

F	π	λ	P
f_1	0.543	0.65	0.7943
f_2	0.457	0.2	0.2057

D	π	λ	P
d_1	0.389	1	0.389
d_2	0.611	1	0.611

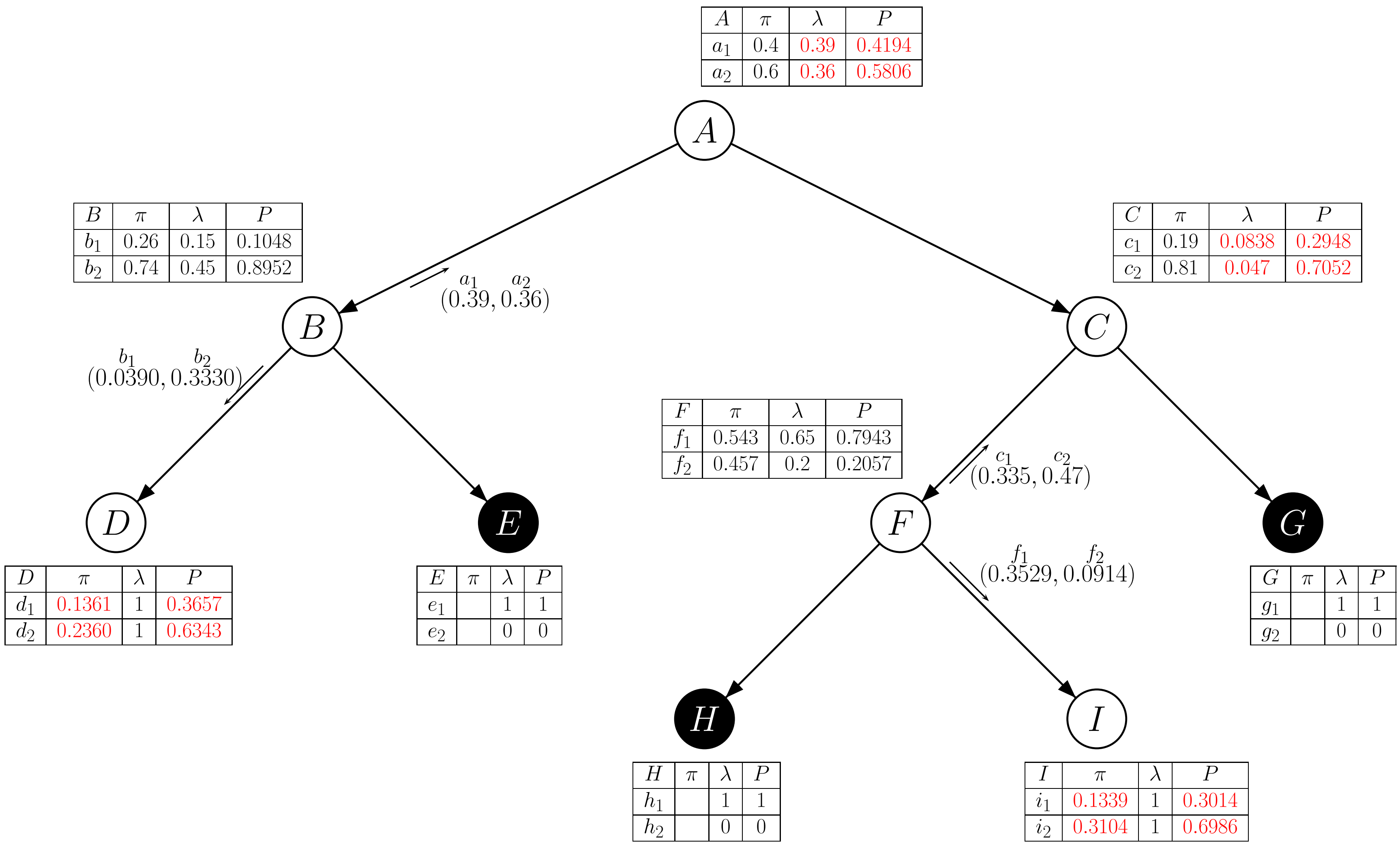
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.3643	1	0.3643
i_2	0.6357	1	0.6357

Larger Network (7): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	0.39	0.4194
a_2	0.6	0.36	0.5806

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.0838	0.2948
c_2	0.81	0.047	0.7052

F	π	λ	P
f_1	0.543	0.65	0.7943
f_2	0.457	0.2	0.2057

D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

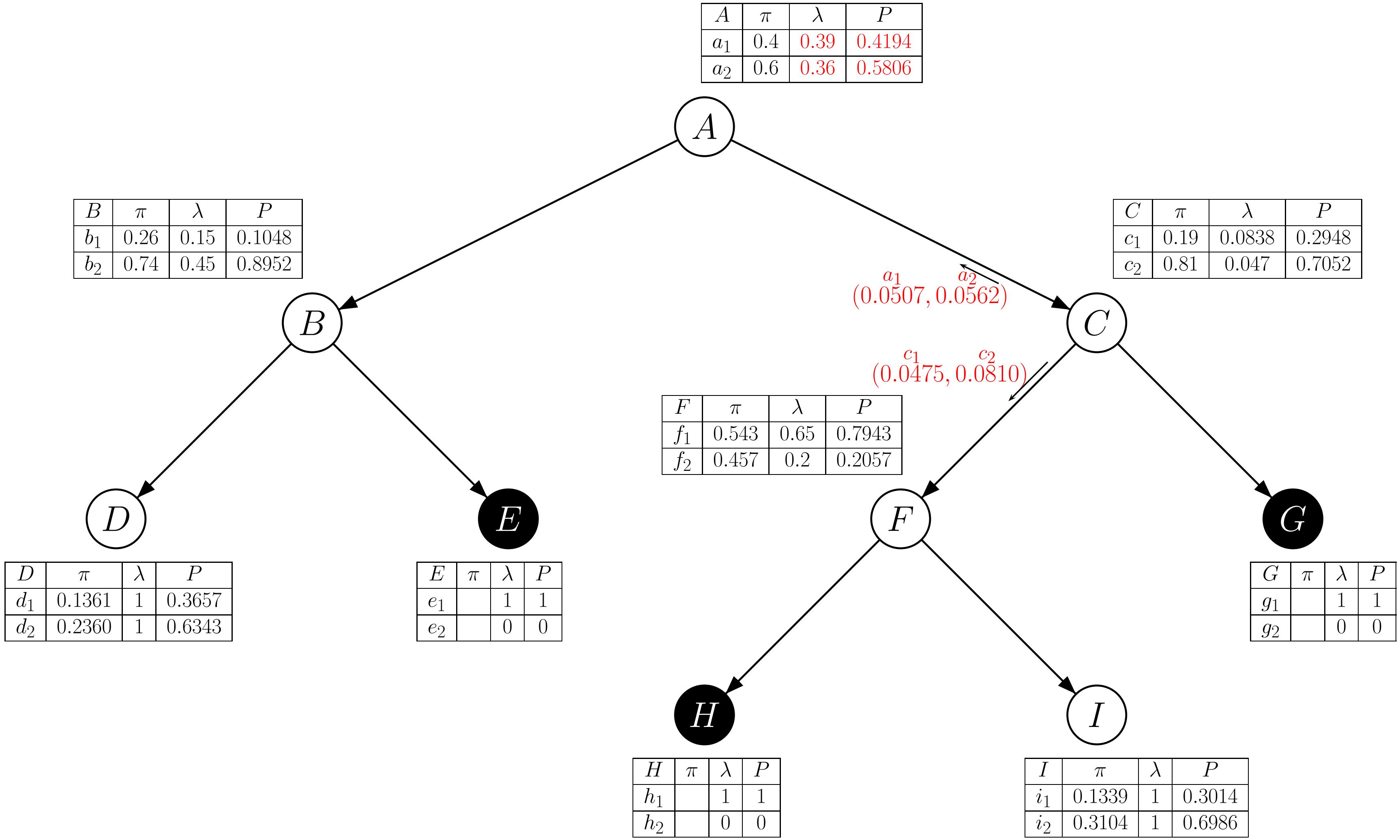
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

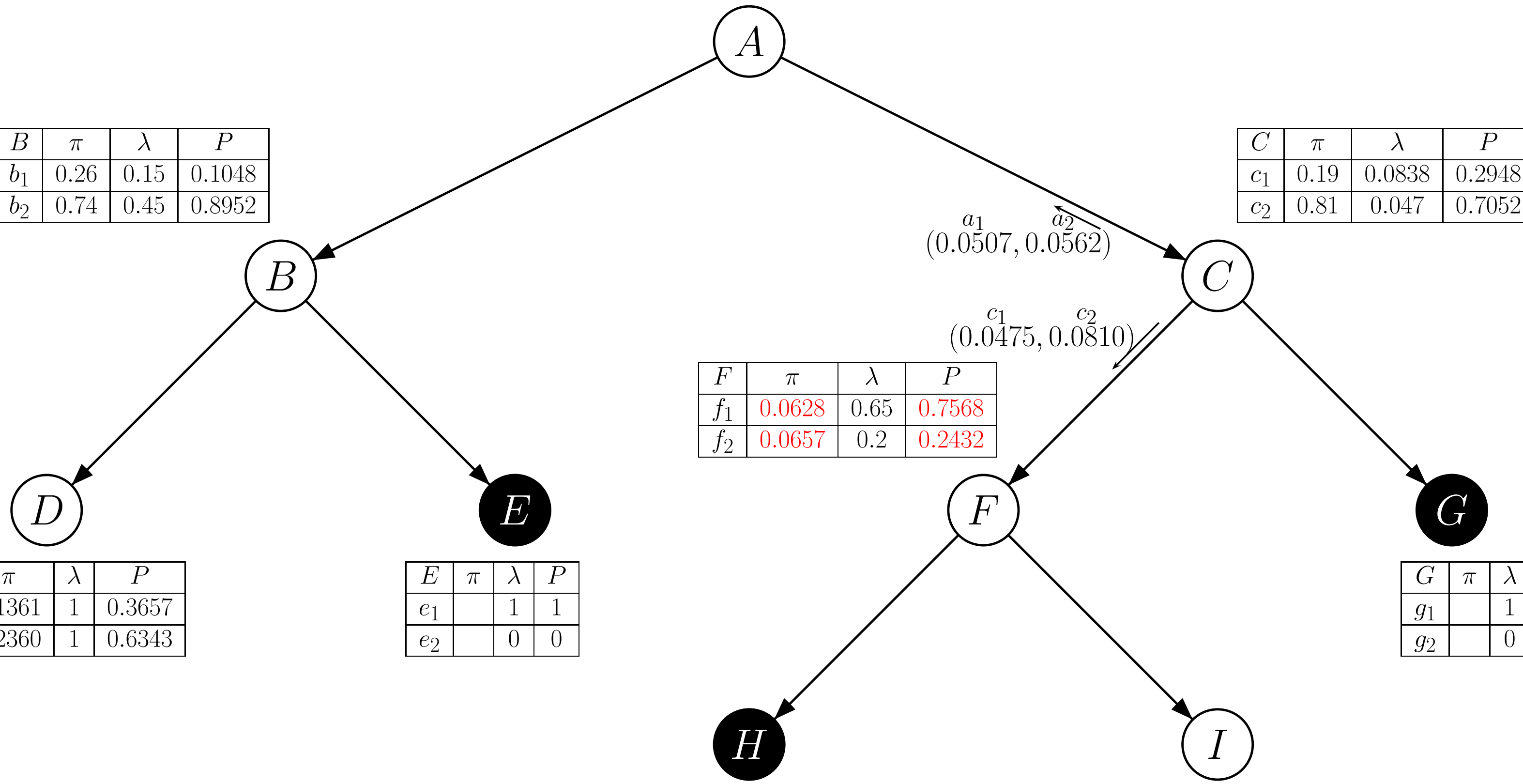
H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.1339	1	0.3014
i_2	0.3104	1	0.6986

Larger Network (8): Propagate Evidence, cont.



Larger Network (9): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.0838	0.2948
c_2	0.81	0.047	0.7052

F	π	λ	P
f_1	0.0628	0.65	0.7568
f_2	0.0657	0.2	0.2432

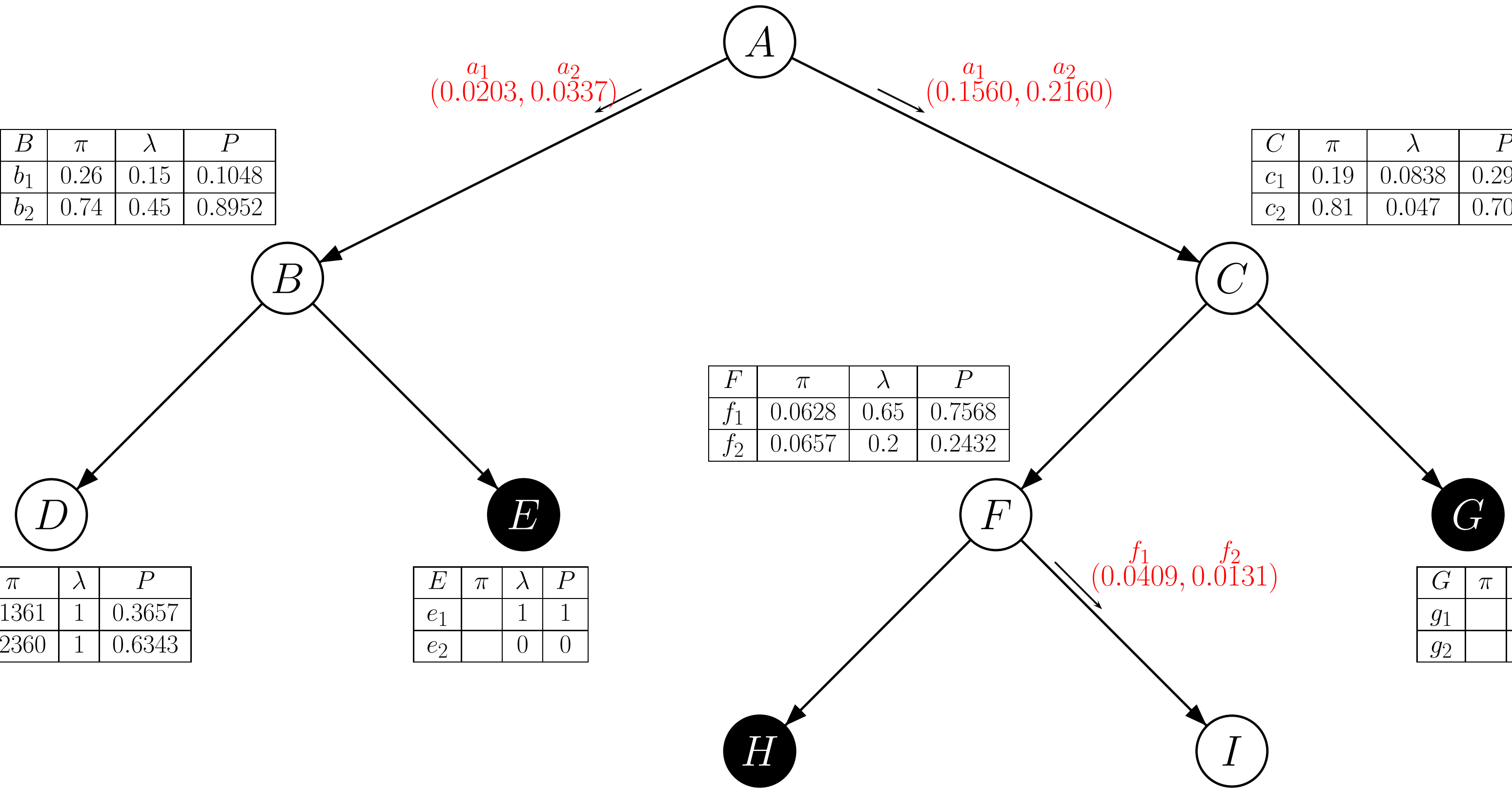
D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.1339	1	0.3014
i_2	0.3104	1	0.6986



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.26	0.15	0.1048
b_2	0.74	0.45	0.8952

C	π	λ	P
c_1	0.19	0.0838	0.2948
c_2	0.81	0.047	0.7052

F	π	λ	P
f_1	0.0628	0.65	0.7568
f_2	0.0657	0.2	0.2432

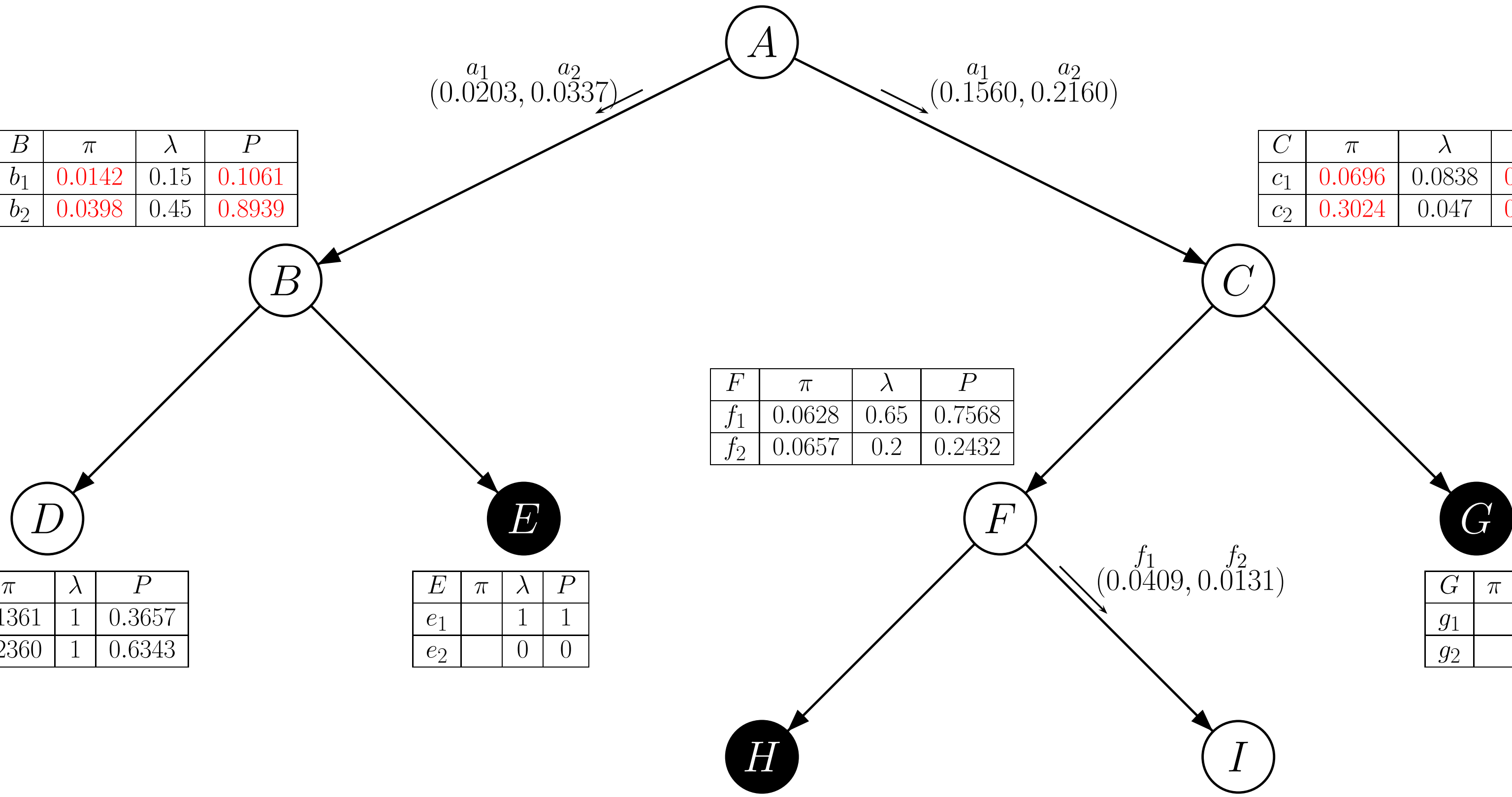
D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.1339	1	0.3014
i_2	0.3104	1	0.6986



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0628	0.65	0.7568
f_2	0.0657	0.2	0.2432

D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

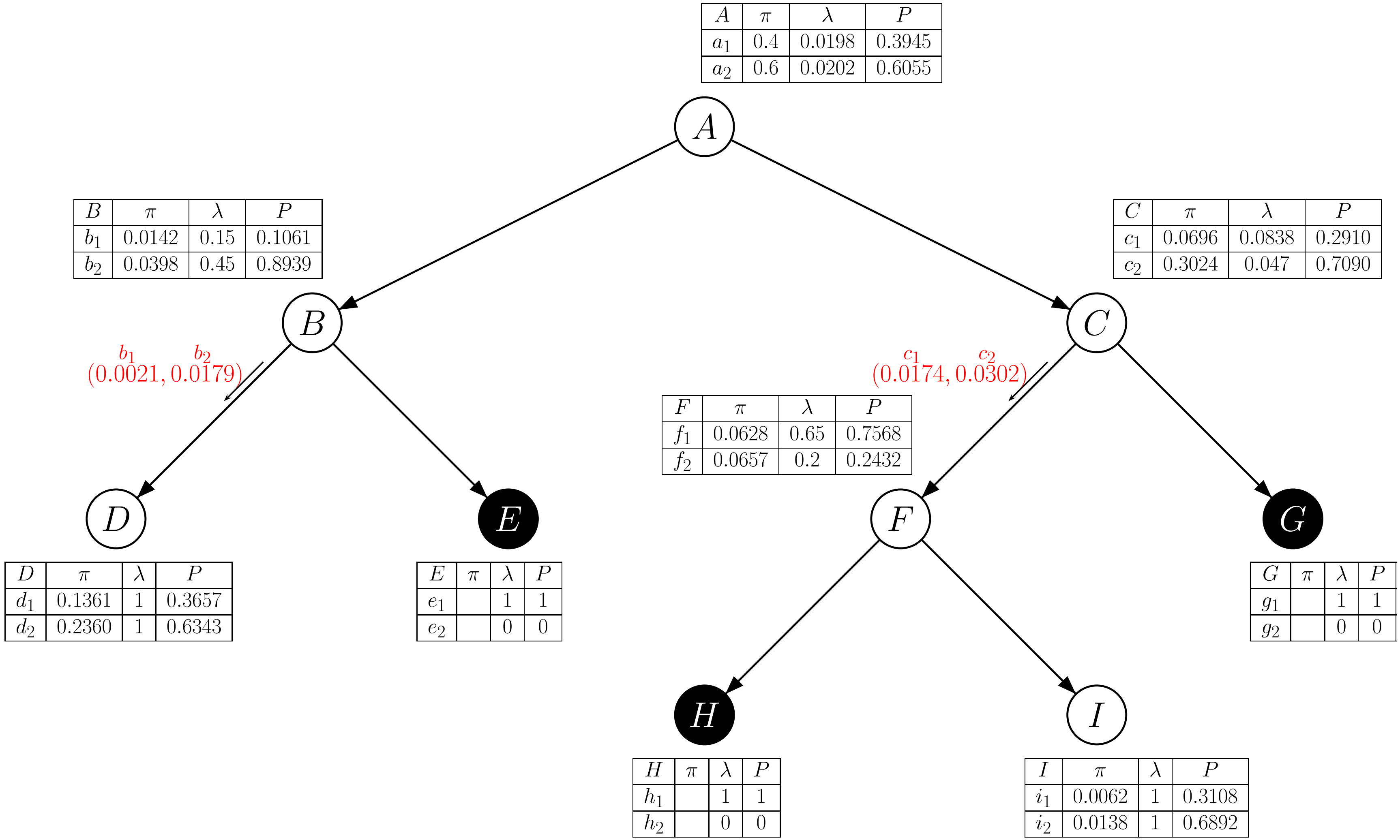
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3108
i_2	0.0138	1	0.6892

Larger Network (12): Propagate Evidence, cont.



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0628	0.65	0.7568
f_2	0.0657	0.2	0.2432

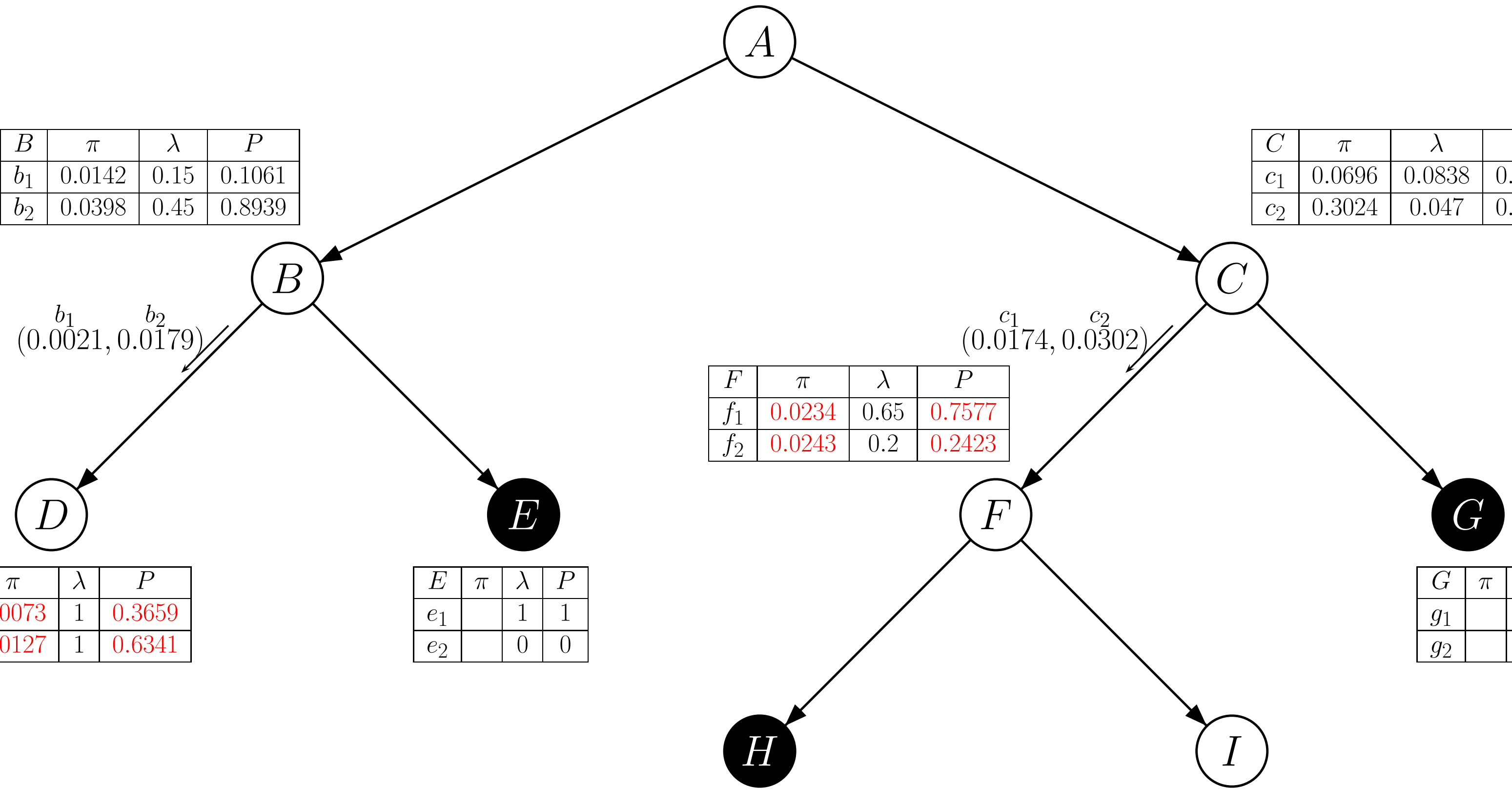
D	π	λ	P
d_1	0.1361	1	0.3657
d_2	0.2360	1	0.6343

E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3108
i_2	0.0138	1	0.6892



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0234	0.65	0.7577
f_2	0.0243	0.2	0.2423

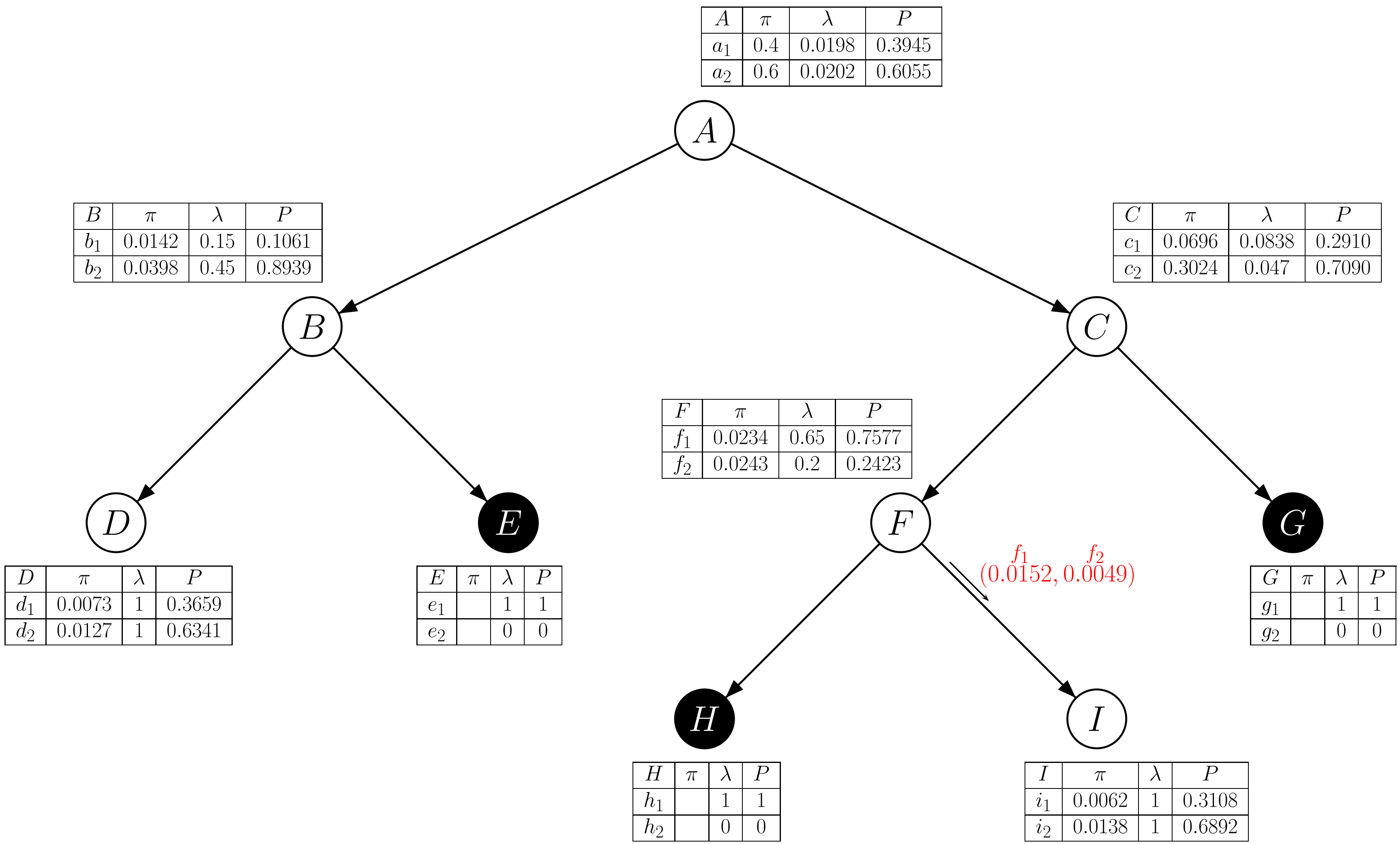
D	π	λ	P
d_1	0.0073	1	0.3659
d_2	0.0127	1	0.6341

E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3108
i_2	0.0138	1	0.6892



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0234	0.65	0.7577
f_2	0.0243	0.2	0.2423

D	π	λ	P
d_1	0.0073	1	0.3659
d_2	0.0127	1	0.6341

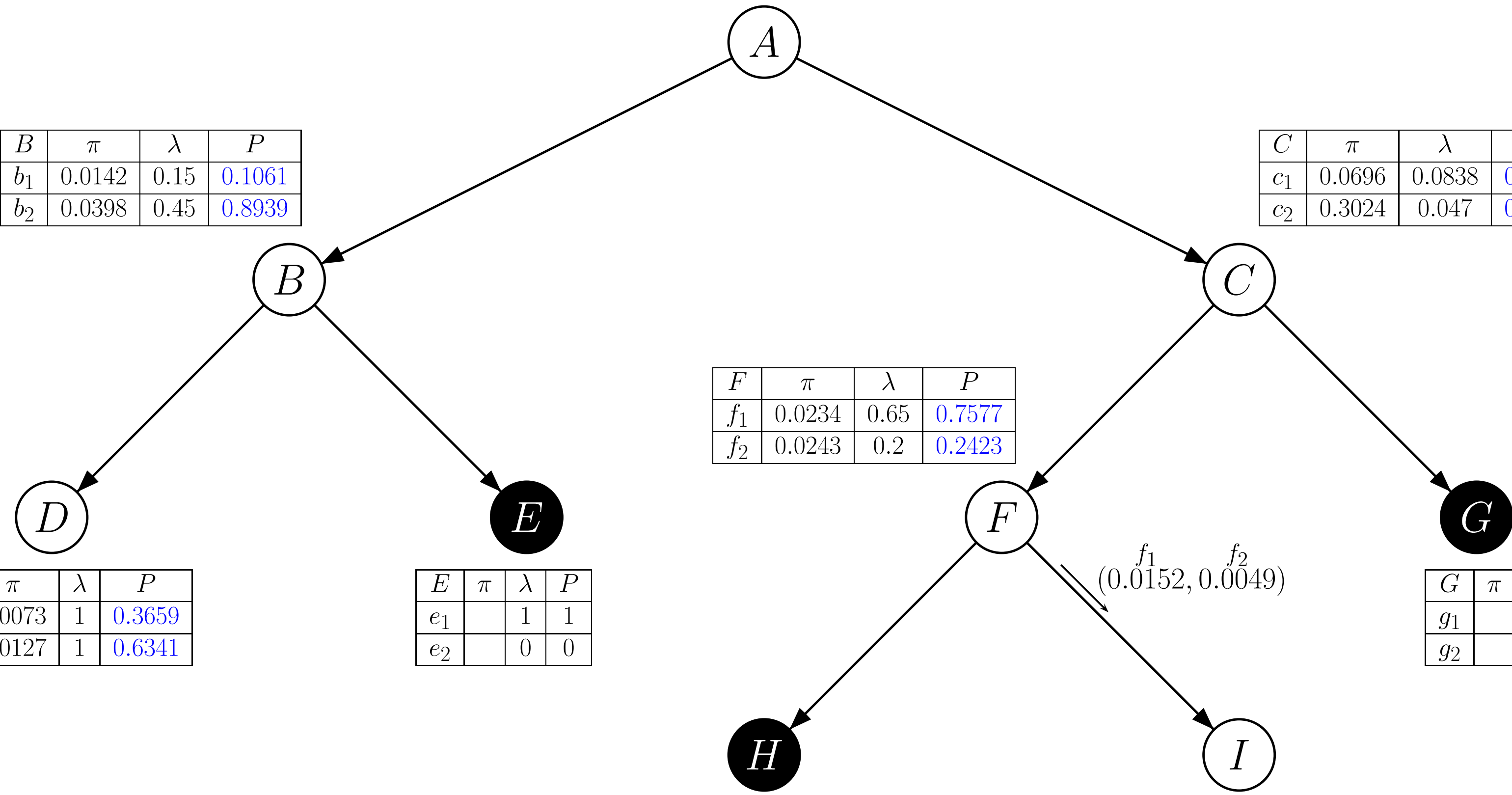
E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3108
i_2	0.0138	1	0.6892

Larger Network (15): Finished



A	π	λ	P
a_1	0.4	0.0198	0.3945
a_2	0.6	0.0202	0.6055

B	π	λ	P
b_1	0.0142	0.15	0.1061
b_2	0.0398	0.45	0.8939

C	π	λ	P
c_1	0.0696	0.0838	0.2910
c_2	0.3024	0.047	0.7090

F	π	λ	P
f_1	0.0234	0.65	0.7577
f_2	0.0243	0.2	0.2423

D	π	λ	P
d_1	0.0073	1	0.3659
d_2	0.0127	1	0.6341

E	π	λ	P
e_1		1	1
e_2		0	0

G	π	λ	P
g_1		1	1
g_2		0	0

H	π	λ	P
h_1		1	1
h_2		0	0

I	π	λ	P
i_1	0.0062	1	0.3106
i_2	0.0138	1	0.6894