# Revision of Bayesian Networks

# Belief Revision in Philosophy

**Belief:** State of mind, that something is the case - often degrees of beliefs are used

There are several different ways for a **Belief Change:**

**Contraction:** removal of a belief;
**Expansion:** addition of a belief without checking consistency;
**Revision:** addition of a belief while maintaining consistency;
**Extraction:** extracting a consistent set of beliefs and/or epistemic entrenchment ordering;
**Consolidation:** restoring consistency of a set of beliefs;
**Merging:** fusion of two or more sets of beliefs while maintaining consistency.

There are postulates for a **logical revision operator** that a rational operator should satisfy (Gärdenfors 1985). But how to change **Belief degrees**?

Example for a **Revision**: It is known that a certain navigation system can only be included in the car if one of the corresponding radio systems is already installed. It is planned to sell 3000 instead of 1000 navigation systems in the next quartal. How many radio systems of each type should be bought?

# Revision of Probabilistic Graphical Models

Graphical models are efficient for representing domain knowledge. After some time additional observations can change our underlying knowledge of the domain.

We need a way to incorporate belief changes to avoid updating the whole knowledge base.

Idea: Local changes should only lead to local adaptations of the knowledge base and neccessary consequences.

# Revision of Probabilistic Graphical Models

Prior Probability Distribution

New conditional probabilities

**Revision**

**necessary** changes, but **minimal** changes

Posterior Probability Distribution including specified and inferred changes

Solution: Search for the Information-theoretically closest distribution to the prior distribution that satisfies the new knowledge

Iterative proportional fitting (raking, matrix scaling), is a well-known algorithm (1937, often reinvented) for adapting the marginal distributions of a given joint distribution to desired values.

It consists in computing the following sequence of probability distributions:

$$p_U^{(0)}(u) \equiv p_U(u) \tag{1}$$

$$\forall 1, 2, \cdots : p_U^{(i)}(u) \equiv p_U^{(i-1)}(u) \frac{p_{A_j}(a)}{p_{A_j}^{(i-1)}(a)} \tag{2}$$

In each step the probability distribution is modified in such a way that the resulting distribution satisfies the given marginal distribution $A_j$. However, this will, in general, change the marginal distribution for an earlier adapted variable $A_k$.

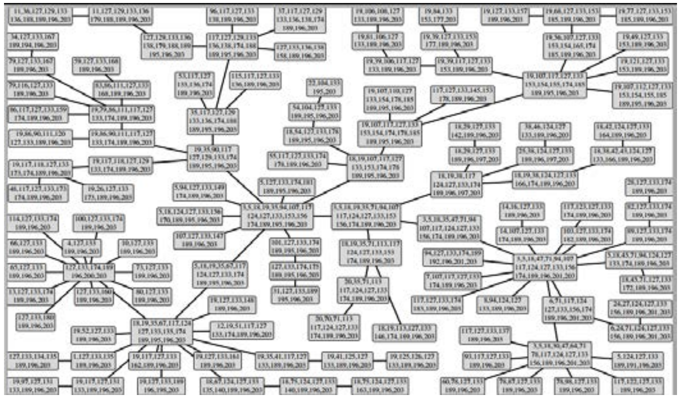
Therefore, the adaptation has to be iterated, traversing the set of variables several times. The process is proofed to converge for non-contradicting revision statements, and **only for statements with no inconsistencies**.

# Revision Algorithm

The revision algorithm sums up as follows:

**forall** $C \in \mathcal{C}$ **do**
    $p_C^{(0)}(c) \equiv p_C(c)$
$i \equiv 0$
**repeat**
$i \equiv i + 1;$
    **forall** $C \in \mathcal{C}$ **do**
        **forall** $j \in J_C$ **do**
            $p_C^{(i)}(c) \equiv p_C^{(i-1)}(c) \frac{p_{A_j}(a)}{p_{A_j}^{(i-1)}(a)};$
        **do** evidence propagation
    **end**
**until** convergence

See https://en.wikipedia.org/wiki/Iterative_proportional_fitting for more details



Inconsistencies can emerge in the presence of:

- **Complex structure:** dependencies between attributes
  e.g. dependencies between car components
- **Many revision assignments:** changes of the probability distribution
  e.g. changing installation rate of component combinations

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 3450 | | | St. | ✓ | 3C -... | 100001 | HOCH |
| | | 3450 | | | St. | ✓ | 3C -... | 100002 | HOCH |
| | (Bereich:EXPORT) | 2729 | | | St. | ✓ | 3C -... | 100003 | HOCH |
| | (Bereich:INLAND) | 721 | | | St. | ✓ | 3C -... | 100004 | HOCH |
| (Bereich:EXPORT) | (Land:ARGENTI... | 18 | | | St. | ✓ | 3C -... | 100005 | HOCH |
| (Bereich:EXPORT) | (Land:AUSTRAL... | 43 | | | St. | ✓ | 3C -... | 100006 | HOCH |
| (Bereich:EXPORT) | (Land:BALTIKUM) | 2 | | | St. | ✓ | 3C -... | 100007 | HOCH |
| (Bereich:EXPORT) | (Land:BELGIEN) | 188 | | | St. | ✓ | 3C -... | 100008 | HOCH |
| (Bereich:EXPORT) | (Land:BRASILIEN) | 15 | | | St. | ✓ | 3C -... | 100009 | HOCH |
| (Bereich:EXPORT) | (Land:BULGARI... | 2 | | | St. | ✓ | 3C -... | 100010 | HOCH |
| (Bereich:EXPORT) | (Land:CHINA) | 0 | | | St. | ✓ | 3C -... | 100011 | HOCH |
| (Bereich:EXPORT) | (Land:DAENEM... | 47 | | | St. | ✓ | 3C -... | 100012 | HOCH |
| (Bereich:EXPORT) | (Land:EXPRESS) | 0 | | | St. | ✓ | 3C -... | 100013 | HOCH |
| (Bereich:EXPORT) | (Land:EXPRESS\... | 0 | | | St. | ✓ | 3C -... | 100014 | HOCH |
| (Bereich:EXPORT) | (Land:EXPRESS\... | 0 | | | St. | ✓ | 3C -... | 100015 | HOCH |
| (Bereich:EXPORT) | (Land:EXPRESS\... | 0 | | | St. | ✓ | 3C -... | 100016 | HOCH |
| (Bereich:EXPORT) | (Land:EXPRESS\... | 0 | | | St. | ✓ | 3C -... | 100017 | HOCH |
| (Bereich:EXPORT) | (Land:EXPRESS\... | 0 | | | St. | ✓ | 3C -... | 100018 | HOCH |
| (Bereich:EXPORT) | (Land:FINNLAND) | 26 | | | St. | ✓ | 3C -... | 100019 | HOCH |
| (Bereich:EXPORT) | (Land:FRANKRE... | 230 | | | St. | ✓ | 3C -... | 100020 | HOCH |
| (Bereich:EXPORT) | (Land:GRIECHE... | 7 | | | St. | ✓ | 3C -... | 100021 | HOCH |
| (Bereich:EXPORT) | (Land:GROSSBR... | 978 | | | St. | ✓ | 3C -... | 100022 | HOCH |
| (Bereich:EXPORT) | (Land:HONGKO... | 8 | | | St. | ✓ | 3C -... | 100023 | HOCH |
| (Bereich:EXPORT) | (Land:INDIEN) | 0 | | | St. | ✓ | 3C -... | 100024 | HOCH |
| (Bereich:EXPORT) | (Land:IRLAND) | 7 | | | St. | ✓ | 3C -... | 100025 | HOCH |
| (Bereich:EXPORT) | (Land:ISLAND) | 0 | | | St. | ✓ | 3C -... | 100026 | HOCH |
| (Bereich:EXPORT) | (Land:ISRAEL) | 6 | | | St. | ✓ | 3C -... | 100027 | HOCH |
| (Bereich:EXPORT) | (Land:ITALIEN) | 265 | | | St. | ✓ | 3C -... | 100028 | HOCH |
| (Bereich:EXPORT) | (Land:JAPAN) | 96 | | | St. | ✓ | 3C -... | 100029 | HOCH |
| (Bereich:EXPORT) | (Land:KANADA) | 0 | | | St. | ✓ | 3C -... | 100030 | HOCH |
| (Bereich:EXPORT) | (Land:KANAREN) | 2 | | | St. | ✓ | 3C -... | 100031 | HOCH |
| (Bereich:EXPORT) | (Land:KROATIEN) | 3 | | | St. | ✓ | 3C -... | 100032 | HOCH |
| (Bereich:EXPORT) | (Land:LUXEMBU... | 13 | | | St. | ✓ | 3C -... | 100033 | HOCH |
| (Bereich:EXPORT) | (Land:MALAYSIA) | 0 | | | St. | ✓ | 3C -... | 100034 | HOCH |
| (Bereich:EXPORT) | (Land:MEXIKO) | 0 | | | St. | ✓ | 3C -... | 100035 | HOCH |
| (Bereich:EXPORT) | (Land:NEUSEEL... | 7 | | | St. | ✓ | 3C -... | 100036 | HOCH |
| (Bereich:EXPORT) | (Land:NIEDERL... | 95 | | | St. | ✓ | 3C -... | 100037 | HOCH |
| (Bereich:EXPORT) | (Land:NORWEG... | 39 | | | St. | ✓ | 3C -... | 100038 | HOCH |
| (Bereich:EXPORT) | (Land:O\ AFRIKA) | 3 | | | St. | ✓ | 3C -... | 100039 | HOCH |
| (Bereich:EXPORT) | (Land:O\-EURO... | 1 | | | St. | ✓ | 3C -... | 100040 | HOCH |
| (Bereich:EXPORT) | (Land:OESTERR... | 65 | | | St. | ✓ | 3C -... | 100041 | HOCH |
| (Bereich:EXPORT) | (Land:POLEN) | 10 | | | St. | ✓ | 3C -... | 100042 | HOCH |
| (Bereich:EXPORT) | (Land:PORTUGAL) | 25 | | | St. | ✓ | 3C -... | 100043 | HOCH |
| (Bereich:EXPORT) | (Land:R\ AGCC) | 0 | | | St. | ✓ | 3C -... | 100044 | HOCH |
| (Bereich:EXPORT) | (Land:R\ AS\-PAZ) | 1 | | | St. | ✓ | 3C -... | 100045 | HOCH |
| (Bereich:EXPORT) | (Land:R\ ASEAN) | 0 | | | St. | ✓ | 3C -... | 100046 | HOCH |
| (Bereich:EXPORT) | (Land:R\ NAHOST) | 0 | | | St. | ✓ | 3C -... | 100047 | HOCH |
| (Bereich:EXPORT) | (Land:R\ SUEDAM) | 13 | | | St. | ✓ | 3C -... | 100048 | HOCH |
| (Bereich:EXPORT) | (Land:RUMAENI... | 3 | | | St. | ✓ | 3C -... | 100049 | HOCH |
| (Bereich:EXPORT) | (Land:RUSSLAND) | 61 | | | St. | ✓ | 3C -... | 100050 | HOCH |
| (Bereich:EXPORT) | (Land:S\ AFRIKA) | 42 | | | St. | ✓ | 3C -... | 100051 | HOCH |
| (Bereich:EXPORT) | (Land:SAUDI\ A... | 0 | | | St. | ✓ | 3C -... | 100052 | HOCH |
| (Bereich:EXPORT) | (Land:SCHWED... | 69 | | | St. | ✓ | 3C -... | 100053 | HOCH |
| (Bereich:EXPORT) | (Land:SCHWEIZ) | 64 | | | St. | ✓ | 3C -... | 100054 | HOCH |

Inconsistencies are unincorporatable changes / inconsistent revision assignments

**Inner Inconsistencies**

Revision assignments are inconsistent independent of prior distribution

**Outer Inconsistencies**

Revision assignments inconsistent with zero-values in prior distribution

# Example: Inner Inconsistencies

Insert revision assignments in probability distribution:

| 0.6 | 0.3 | | |
|---|---|---|---|
| 0.2 | 0.25 | | |
| 0.3 | | | 0.5 |
| | | 0.1 | |

Inner inconsistencies can emerge as consequences of probability implications:

| 0.6 | 0.3 | 0.1 | | $\leftarrow 0.10 = 1 - 0.6 - 0.3$ |
|---|---|---|---|---|
| 0.2 | 0.25 | | | |
| 0.3 | | | 0.5 | |
| 0.1 | | 0.1 | | $\leftarrow 0.10 = 0.6 - 0.2 - 0.3$ |

# Example: Inner Inconsistencies

Insert revision assignments in probability distribution:

| 0.6 | 0.3 | | |
|---|---|---|---|
| 0.2 | 0.25 | | |
| 0.3 | | | 0.5 |
| | | 0.1 | |

Inner inconsistencies can emerge as consequences of probability implications:

| 0.6 | 0.3 | 0.1 | | |
|---|---|---|---|---|
| 0.2 | 0.25 | 0.0 | | ← set to zero since column sum is already maximum |
| 0.3 | | 0.0 | 0.5 | ← set to zero since column sum is already maximum |
| 0.1 | | 0.1 | | |

Insert revision assignments in probability distribution:

| 0.6 | 0.3 | | |
|---|---|---|---|
| 0.2 | 0.25 | | |
| 0.3 | | | 0.5 |
| | | 0.1 | |

Inner inconsistencies can emerge as consequences of probability implications:

| 0.6 | 0.3 | 0.1 | |
|---|---|---|---|
| 0.2 | 0.25 | 0.0 | 0.45 |
| 0.3 | | 0.0 | 0.5 |
| 0.1 | | 0.1 | 0.05 |

$\leftarrow \quad 0.45 = 0.2 + 0.25 + 0.0$

# Example: Inner Inconsistencies

Insert revision assignments in probability distribution:

| 0.6 | 0.3 | | |
|---|---|---|---|
| 0.2 | 0.25 | | |
| 0.3 | | | 0.5 |
| | | 0.1 | |

Inner inconsistencies can emerge as consequences of probability implications:

| 0.6 | 0.3 | 0.1 | |
|---|---|---|---|
| 0.2 | 0.25 | 0.0 | 0.45 |
| 0.3 | | 0.0 | 0.5 |
| 0.1 | | 0.1 | ? |

Contradicting implications: $0.05 \neq 0.20$

column-sum $\Rightarrow$ $1 - 0.45 - 0.5 = 0.05$

row-sum $\Rightarrow$ $0.1 + 0.1 = 0.20$

# Example: Outer Inconsistencies

Insert revision assignments in probability distribution with fixed zero values ( × ):

| 0.1 | 0.5 | 0.4 | |
|---|---|---|---|
| | × | × | 0.3 |
| | × | | 0.5 |
| | | | |

Outer inconsistencies can emerge as consequences of probability implications:

| 0.1 | 0.5 | 0.4 | |
|---|---|---|---|
| | × | × | 0.3 |
| | × | | 0.5 |
| | 0.5 | | 0.2 |

Contradicting implications: $0.5 > 0.2$

column-sum $\Rightarrow$ $0.5 - 0.0 - 0.0 = 0.5$

column-sum $\Rightarrow$ $1.0 - 0.3 - 0.5 = 0.2$

# Systematic Handling of Revision Inconsistencies

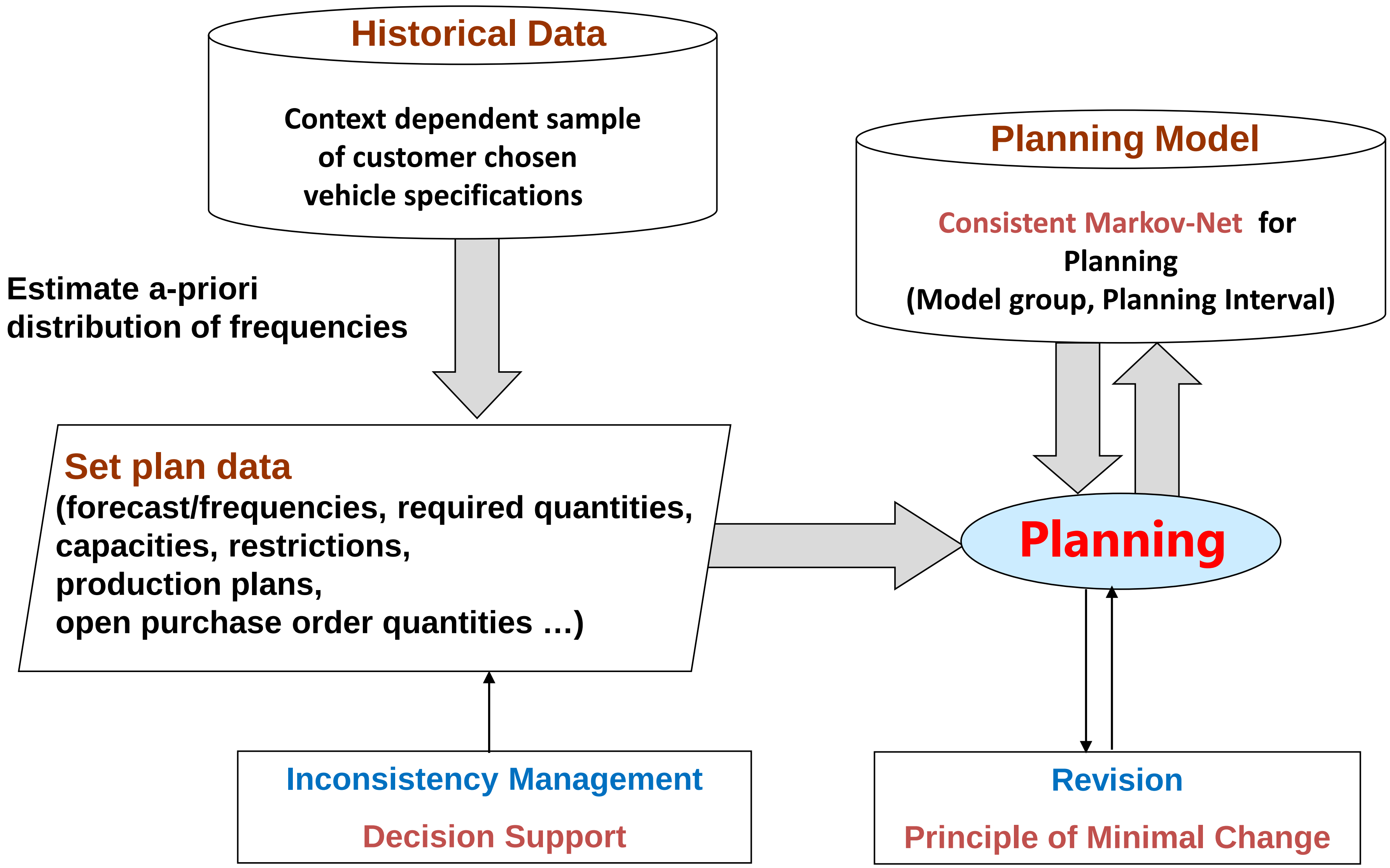
Even for an expert user it is not easy to configure revision statements without creating inconsistencies!

In the case of belief without uncertainty there are methods to handle inconsistencies: The **Logics** of Formal **Inconsistency** (LFIs) are a family of paraconsistent **logics** that constitute consistent fragments of classical **logic** yet which reject the explosion principle where a **contradiction** is present.

The case of graded belief is more complicated:
If the Revision-Operation fails, we need to explain the user how to change his desired revision statements. Otherwise no solution can be found.

There must be an inconsistency management before using the revision operator. In our real example it looks as follows:

# Example: Planning Operation Revision



# Systematic Handling of Revision Inconsistencies

## Example for one planning week

| Data volume (cumulative per week) | Across all 166 Model groups | Single Model group |
|---|---|---|
| Number of Markov Nets | 75.787 | 1.054 |

| … for one planning interval | Model group 1 | Model group 2 |
|---|---|---|
| Planning requirement | 4.424 | 1.299 |
| Number of variables | 203 | 204 |
| Number of cliques | 174 | 156 |
| | | |
| Largest clique (only positive probabilities) | 130.806 tupels<br>9 variables | 1.489.515 tupels<br>14 variables |