Fuzzy Systems
Prof. Dr. Rudolf Kruse, Alexander Dockhorn

Assignment Sheet 10

Assignment 1 Combination of Conjunction and Disjunction

Show that the following pairs of a t-norm $\top$ and a t-conorm $\perp$ are dual *w.r.t.* the standard fuzzy negation $\sim a = 1 - a$:

a) $\top_{\min}(a, b) = \min\{a, b\}$ and $\perp_{\max}(a, b) = \max\{a, b\}$

b) $\top_{\text{prod}}(a, b) = a \cdot b$ and $\perp_{\text{sum}}(a, b) = a + b - ab$

c) $\top_{\text{Łuka}}(a, b) = \max\{a + b - 1,\ 0\}$ and $\perp_{\text{Łuka}}(a, b) = \min\{a + b,\ 1\}$.

(Reminder: A t-norm $\top$ and a t-conorm $\perp$ are called dual *w.r.t.* a negation $\sim$ if the fuzzy analogs of De Morgan's laws are satisfied.)

Assignment 2 Fuzzy Arithmetic

Consider the fuzzy numbers $\mu_{about\_two} = \mu_1$ and $\mu_{about\_one\_hundred} = \mu_2$ defined by

$$\mu_1(x) = \begin{cases} x - 1 & \text{if } 1 \leq x \leq 2, \\ 3 - x & \text{if } 2 \leq x \leq 3, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad \mu_2(x) = \begin{cases} \frac{1}{5}(x - 95) & \text{if } 95 \leq x \leq 100, \\ \frac{1}{5}(105 - x) & \text{if } 100 \leq x \leq 105, \\ 0 & \text{otherwise.} \end{cases}$$

Determine an explicit form of the fuzzy sets $\mu_1 + \mu_2$, $\mu_1 \cdot \mu_2$, and $50 \cdot \mu_1 - \mu_2$ using a set representation of the fuzzy numbers.

Assignment 3 Fuzzy Relational Equations

Let $X = \{1, 2, 3\}$ and $Y = \{a, b\}$. Consider the fuzzy sets μ_1, μ_2, μ_3 on X and ν_1, ν_2, ν_3 on Y which are defined as shown below. Be $\mu_i \circ \varrho = \nu_i$ for all $i = 1, 2, 3$ a system of fuzzy relational equations.

$\mu_i(x)$	1	2	3
μ_1	0.1	0.3	0.7
μ_2	0.9	0.6	0.3
μ_3	0.5	0.2	0.0

$\nu_i(y)$	a	b
ν_1	0.6	0.2
ν_2	0.3	0.6
ν_3	0.1	0.5

a) Find the greatest solution of this system.

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Assignment 4 Automatic Height Control

Design a Mamdani-Assilian Controller which automatically holds the height for an hot-air balloon. The goal is to stabilize a height of 1000 meters above the ground as long as there is no vertical turbulence. When such an aerial turbulence occurs it shall increase the height for safety reasons till maximally 2500 meters. This can be performed by controlling two valves, the first drains hot air (to reduce the ascending force), the second regulates the amount of gas, burned to heat the air within the balloon (increasing the ascending force). The balloon is equipped with sensors which determine the height and the vertical force (for measuring the turbulence). For simplicity we only care about the vertical dimension.

a) Define linguistic variables to describe the necessary variables. Sketch the fuzzy partition for each linguistic variable by choosing appropriate fuzzy sets.
b) Suggest operations to compute the rule activations, to combine outputs of simultaneously active rules, and to defuzzify the final output fuzzy set.
c) Specify a set of rules that is suitable to control the vehicle.
d) Which conditions should be satisfied regarding the control variables?