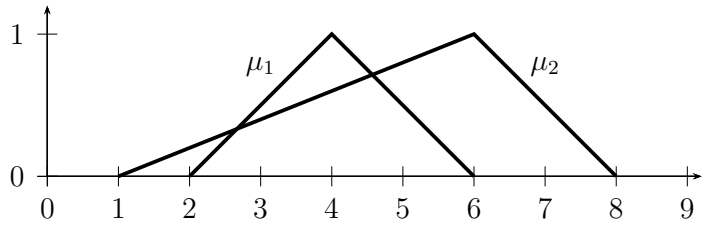
Fuzzy Systems
Prof. Dr. Rudolf Kruse, Alexander Dockhorn

Assignment Sheet 7

Assignment 24 Fuzzy Set Operations

Let the following two fuzzy sets be given:



Compute and draw

a) the intersection of μ_1 and μ_2 using the algebraic product $\top_{\text{prod}}$,

b) the union of μ_1 and μ_2 using the algebraic sum $\perp_{\text{sum}}$,

c) the degree to which μ_1 is a subset of μ_2 using the Łukasiewicz implication.

Assignment 25 Fuzzy Relational Equations

Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3, y_4\}$ be two sets and $\mu : X \to [0, 1]$ and $\nu : Y \to [0, 1]$ two fuzzy sets on X and Y, respectively, which are defined as follows:

$$\begin{array}{llll} \mu(x_1) = 0.1, & \mu(x_2) = 0.7, & \mu(x_3) = 1.0, & \\ \nu(y_1) = 0.4, & \nu(y_2) = 1.0, & \nu(y_3) = 0.8, & \nu(y_4) = 0.3. \end{array}$$

a) How can you find out whether the relational equation $\mu \circ \varrho = \nu$ has a solution, *i.e.* whether there is a fuzzy relation ϱ that satisfies this equation?

b) If the relational equation $\mu \circ \varrho = \nu$ has a solution, determine a solution. Are there other solutions than the one you found?

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Assignment 26 Fuzzy Relational Equations

Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ be two sets. Consider the fuzzy sets μ_1, μ_2, μ_3 on X and ν_1, ν_2, ν_3 on Y which are defined as shown in the two tables below.

	x_1	x_2	x_3
μ_1	1.0	0.6	0.2
μ_2	0.0	0.8	1.0
μ_3	0.9	0.1	0.0

	y_1	y_2
ν_1	1.0	0.4
ν_2	0.6	1.0
ν_3	0.9	0.5

a) Show that the system consisting of the two relational equations $\mu_1 \circ \varrho = \nu_1$ and $\mu_2 \circ \varrho = \nu_2$ has a solution. Find the greatest solution of this system.

b) Is the fuzzy relation that can be computed as the union (maximum) of the two Cartesian products $\mu_1 \otimes \nu_1$ and $\mu_2 \otimes \nu_2$ also a solution of the system of relational equations considered in a)?

c) Show that the system consisting of the three relational equations $\mu_i \circ \varrho = \nu_i$, $i = 1, 2, 3$, does not have any solution.

Assignment 27 System of Relational Equations

Saint Nicholas has somewhat read the manual of his Robopet in the meantime. To tryout what has been learned already, he wants to teach the pet how to greet the reindeers and him. Of course, the pet shall be responsive much stronger to Nicholas than to the reindeers. The recognition shall be performed with embedded color sensors. They measure how similar incoming light is to the respective elementary colors.

Saint Nicholas uses the basic set $C = \{r, g, b\}$ (red, green, blue) and $E = \{a, p, j\}$ (attention, protective instinct, joy). Furthermore, he defines fuzzy sets $\mu_1 : C \to [0, 1]$, $\mu_2 : C \to [0, 1]$ and $\nu_1 : E \to [0, 1]$, $\nu_2 : E \to [0, 1]$ as shown in the two tables below.

	r	g	b
μ_1	1.0	0.1	0.1
μ_2	0.5	0.2	0.3

	a	p	j
ν_1	0.2	0.6	1.0
ν_2	0.2	1.0	0.2

Finally, he creates a controller based on Gödel's implication with the following rules

if c **is** μ_1 **then** e **is** ν_1**,**
if c **is** μ_2 **then** e **is** ν_2**,**

where $c \in C$ and $e \in E$.

a) Why might Nicholas not be fully satisfied with the result? How can he avoid this problem?

b) Use the Gödel relation to compute the fuzzy output value of the controller when Robopet sees the waste collection vehicle with $\mu_1(r) = \mu_1(g) = \mu_1(b) = 0.6$.